What's our vector, Victor?

— Captain Oveur in *Airplane!* (1980)

6. Normalize the following vectors:

   (a) $\begin{bmatrix}12 & 5\end{bmatrix}$

   (b) $\begin{bmatrix}0 & 743.632\end{bmatrix}$

   (c) $\begin{bmatrix}8 & -3 & 1/2\end{bmatrix}$

7. Evaluate the following vector expressions:

   (a) $\begin{bmatrix}7 & -2 & -3\end{bmatrix} + \begin{bmatrix}6 & 6 & -4\end{bmatrix}$

   (b) $\begin{bmatrix}2 & 9 & -1\end{bmatrix} + \begin{bmatrix}-2 & -9 & 1\end{bmatrix}$

   (c) $\begin{bmatrix}3 \\ 10 \\ 7\end{bmatrix} - \begin{bmatrix}8 \\ -7 \\ 4\end{bmatrix}$

9. Evaluate the following vector expressions:

   (a) $\begin{bmatrix}2 \\ 6\end{bmatrix} \cdot \begin{bmatrix}-3 \\ 8\end{bmatrix}$

   (b) $-7\begin{bmatrix}1 & 2\end{bmatrix} \cdot \begin{bmatrix}11 & -4\end{bmatrix}$

   (c) $10 + \begin{bmatrix}-5 \\ 1 \\ 3\end{bmatrix} \cdot \begin{bmatrix}4 \\ -13 \\ 9\end{bmatrix}$

   (d) $3\begin{bmatrix}-2 \\ 0 \\ 4\end{bmatrix} \cdot \left(\begin{bmatrix}8 \\ -2 \\ 3/2\end{bmatrix} + \begin{bmatrix}0 \\ 9 \\ 7\end{bmatrix}\right)$

20. A nonplayer character (NPC) is standing at location $\mathbf{p}$ with a forward direction of $\mathbf{v}$.

    (a) How can the dot product be used to determine whether the point $\mathbf{x}$ is in front of or behind the NPC?

    (b) Let $\mathbf{p} = \begin{bmatrix}-3 & 4\end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix}5 & -2\end{bmatrix}$. For each of the following points $\mathbf{x}$ determine whether $\mathbf{x}$ is in front of or behind the NPC:

    (1) $\mathbf{x} = \begin{bmatrix}0 & 0\end{bmatrix}$

    (2) $\mathbf{x} = \begin{bmatrix}1 & 6\end{bmatrix}$

    (3) $\mathbf{x} = \begin{bmatrix}-6 & 0\end{bmatrix}$

    (4) $\mathbf{x} = \begin{bmatrix}-4 & 7\end{bmatrix}$

21. Extending the concept from Exercise 20, consider the case where the NPC has a limited field of view (FOV). If the total FOV angle is ϕ, then the NPC can see to the left or right of its forward direction by a maximum angle of $\phi/2$.

(a) How can the dot product be used to determine whether the point $\mathbf{x}$ is visible to the NPC?

(b) For each of the points $\mathbf{x}$ in Exercise 20 determine whether $\mathbf{x}$ is visible to the NPC if its FOV is $90°$.

(c) Suppose that the NPC's viewing distance is also limited to a maximum distance of 7 units. Which points are visible to the NPC then?

4. Compute the following matrix products. If the product is not possible, just say so.

(a) $\begin{bmatrix} 1 & -2 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} -3 & 7 \\ 4 & 1/3 \end{bmatrix}$

(b) $\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} 3 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -1 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 3 \\ 5 & 7 & -6 \\ 1 & -4 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} x & y & z & w \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

7. Describe the transformation $\mathbf{aM} = \mathbf{b}$ represented by each of the following matrices.

(a) $\mathbf{M} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(b) $\mathbf{M} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

(c) $\mathbf{M} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

(d) $\mathbf{M} = \begin{bmatrix} 4 & 0 \\ 0 & 7 \end{bmatrix}$

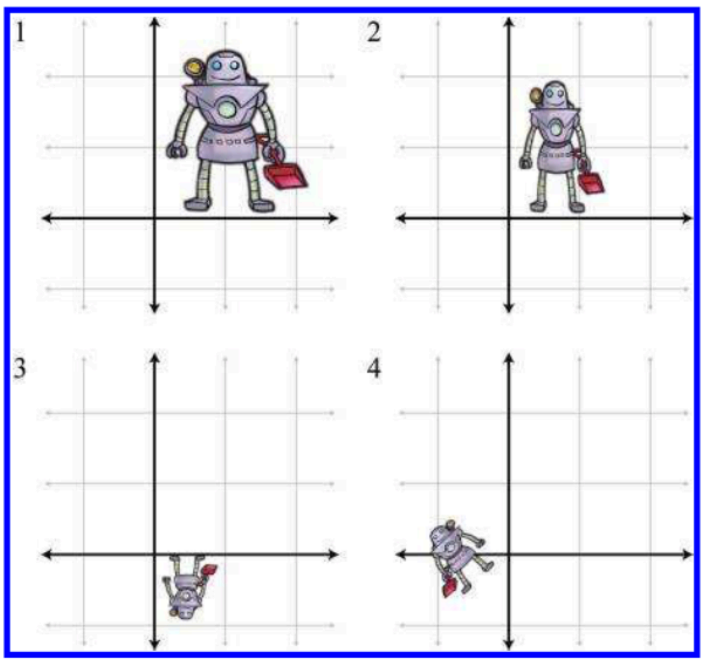
9. Match each of the following figures (1–4) with their corresponding transformations.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 2.5 & 0 \\ 0 & 2.5 \end{bmatrix}$

(c) $\begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

(d) $\begin{bmatrix} 1.5 & 0 \\ 0 & 2.0 \end{bmatrix}$



2. Construct a matrix to rotate $-22°$ about the x-axis.

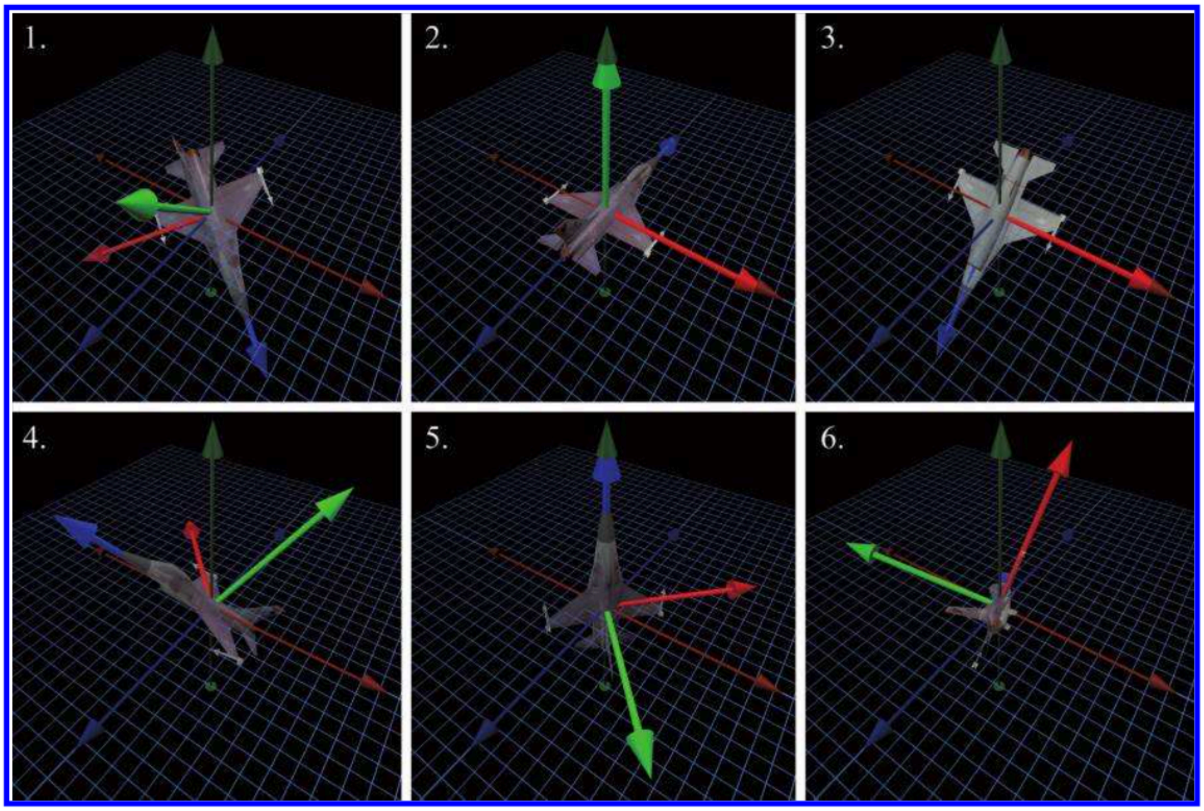


Figure 8.13
Sample orientations used for Exercises 1, 2, 4, and 5.

8.8 Exercises

(Answers on page 772.)

1. Match each of the rotation matrices below with the corresponding orientation from Figure 8.13. These matrices transform row vectors on the left from object space to upright space.

   (a) $\begin{bmatrix} 0.707 & 0.000 & 0.707 \\ 0.707 & 0.000 & -0.707 \\ 0.000 & 1.000 & 0.000 \end{bmatrix}$

   (b) $\begin{bmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & -0.707 & 0.707 \\ 0.000 & -0.707 & -0.707 \end{bmatrix}$