5.48. (a)

Property	Applies?	Comments
Stable	No	For a stable, causal system, all poles must be inside the unit circle.
IIR	Yes	The system has poles at locations other than $z = 0$ or $z = \infty$.
FIR	No	FIR systems can only have poles at $z = 0$ or $z = \infty$.
Minimum Phase	No	Minimum phase systems have all poles and zeros located inside the unit circle.
Allpass	No	Allpass systems have poles and zeros in conjugate reciprocal pairs.
Generalized Linear Phase	No	The causal generalized linear phase systems presented in this chapter are FIR.
Positive Group Delay for all w	No	This system is not in the appropriate form.

(b)

Property	Applies?	Comments
Stable	Yes	The ROC for this system function, $\|z\| > 0$, contains the unit circle. (Note there is 7th order pole at $z = 0$).
IIR	No	The system has poles only at $z = 0$.
FIR	Yes	The system has poles only at $z = 0$.
Minimum Phase	No	By definition, a minimum phase system must have all its poles and zeros located *inside* the unit circle.
Allpass	No	Note that the zeros on the unit circle will cause the magnitude spectrum to drop zero at certain frequencies. Clearly, this system is not allpass.
Generalized Linear Phase	Yes	This is the pole/zero plot of a type II FIR linear phase system.
Positive Group Delay for all w	Yes	This system is causal and linear phase. Consequently, its group delay is a positive constant.

(c)

Property	Applies?	Comments
Stable	Yes	All poles are inside the unit circle. Since the system is causal, the ROC includes the unit circle.
IIR	Yes	The system has poles at locations other than $z = 0$ or $z = \infty$.
FIR	No	FIR systems can only have poles at $z = 0$ or $z = \infty$.
Minimum Phase	No	Minimum phase systems have all poles and zeros located inside the unit circle.
Allpass	Yes	The poles inside the unit circle have corresponding zeros located at conjugate reciprocal locations.
Generalized Linear Phase	No	The causal generalized linear phase systems presented in this chapter are FIR.
Positive Group Delay for all w	Yes	Stable allpass systems have positive group delay for all w.

ECE 844 – Homework 6 Solutions

6.23. Causal LTI system with system function:

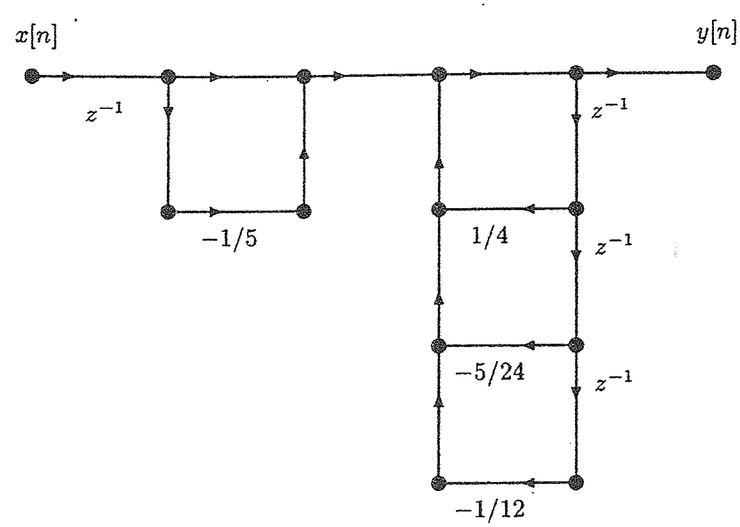
$$H(z) = \frac{1-\frac{1}{5}z^{-1}}{(1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2})(1+\frac{1}{4}z^{-1})}.$$

(a) (i) Direct form I.

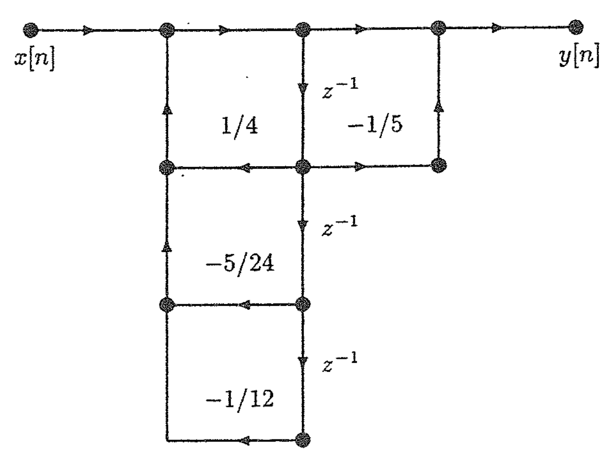
$$H(z) = \frac{1-\frac{1}{5}z^{-1}}{1-\frac{1}{4}z^{-1}+\frac{5}{24}z^{-2}+\frac{1}{12}z(-3)}$$

so

$$b_0 = 1\ ,\ b_1 = -\frac{1}{5} \text{ and } a_1 = \frac{1}{4}\ ,\ a_2 = -\frac{5}{24}\ ,\ a_3 = -\frac{1}{12}.$$



(ii) Direct form II.

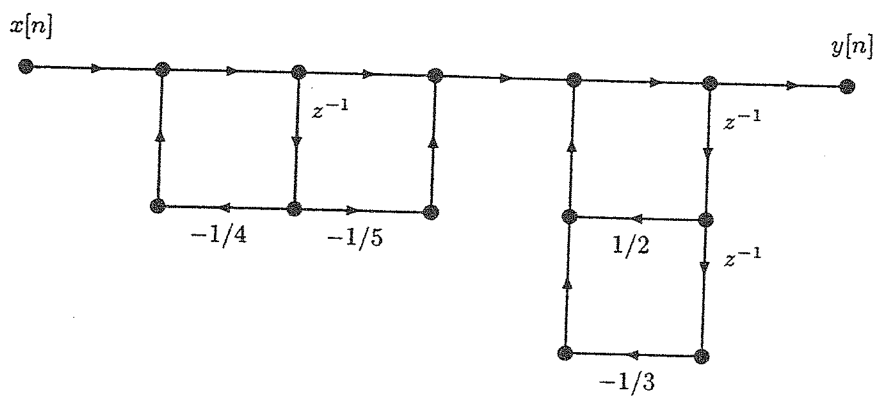


(iii) Cascade form using first and second order direct form II sections.

$$H(z) = \left(\frac{1-\frac{1}{5}z^{-1}}{1+\frac{1}{4}z^{-1}}\right)\left(\frac{1}{1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}}\right).$$

So

$$b_{01} = 1\ ,\ b_{11} = -\tfrac{1}{5}\ ,\ b_{21} = 0\ ,$$
$$b_{02} = 1\ ,\ b_{12} = 0\ ,\ b_{22} = 0 \text{ and}$$
$$a_{11} = -\tfrac{1}{4}\ ,\ a_{21} = 0\ ,\ a_{12} = \tfrac{1}{2}\ ,\ a_{22} = -\tfrac{1}{3}.$$



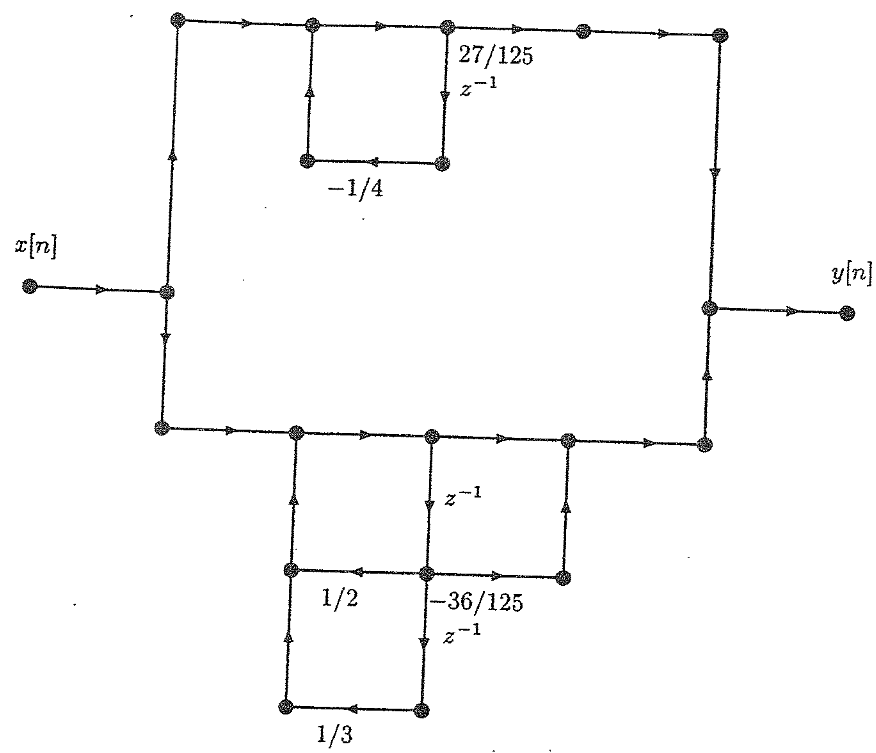
(iv) Parallel form using first and second order direct form II sections.
We can rewrite the transfer function as:

$$H(z) = \frac{\frac{27}{125}}{1+\frac{1}{4}z^{-1}} + \frac{\frac{98}{125} - \frac{36}{125}z^{-1}}{1-\frac{1}{2}z^{-1}-\frac{1}{3}z^{-2}}.$$

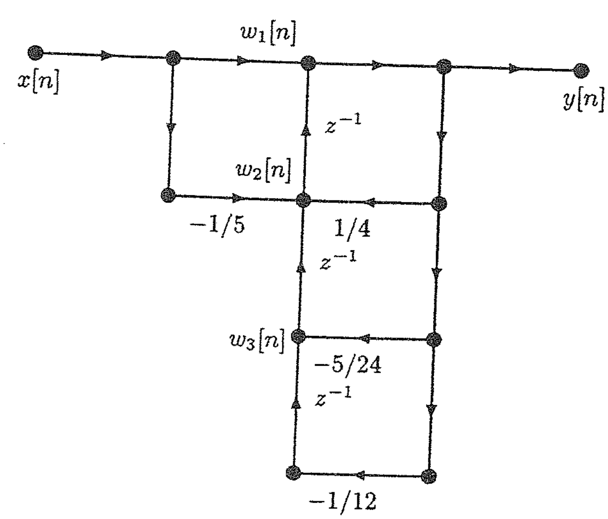
So

$$e_{01} = \tfrac{27}{125}\ ,\ e_{11} = 0\ ,$$
$$e_{02} = \tfrac{98}{125}\ ,\ e_{12} = -\tfrac{36}{125}\ ,\ \text{ and}$$
$$a_{11} = -\tfrac{1}{4}\ ,\ a_{21} = 0\ ,\ a_{12} = \tfrac{1}{2}\ ,\ a_{22} = -\tfrac{1}{3}.$$

3



(v) Transposed direct form II
We take the direct form II derived in part (ii) and reverse the arrows as well as exchange the input and output. Then redrawing the flow graph, we get:



(b) To get the difference equation for the flow graph of part (v) in (a), we first define the intermediate variables: $w_1[n]$, $w_2[n]$ and $w_3[n]$. We have:

$$\begin{aligned}
(1)\ w_1[n] &= x[n] + w_2[n-1] \\
(2)\ w_2[n] &= \frac{1}{4}y[n] + w_3[n-1] - \frac{1}{5}x[n] \\
(3)\ w_3[n] &= -\frac{5}{24}y[n] - \frac{1}{12}y[n-1] \\
(4)\quad y[n] &= w_1[n].
\end{aligned}$$

Combining the above equations, we get:

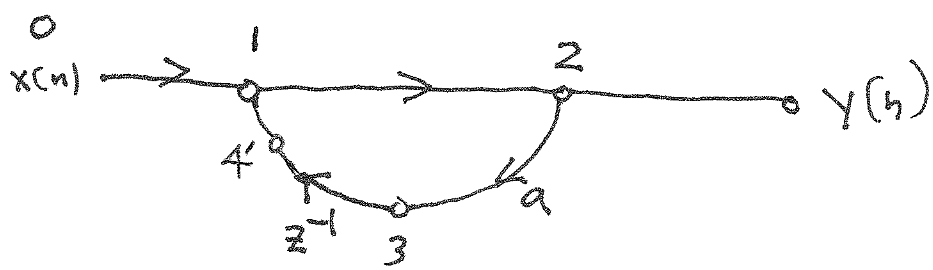
$$y[n] - \frac{1}{4}y[n-1] + \frac{5}{24}y[n-2] + \frac{1}{12}y[n-3] = x[n] - \frac{1}{5}x[n-1].$$

Taking the Z-transform of this equation and combining terms, we get the following transfer function:

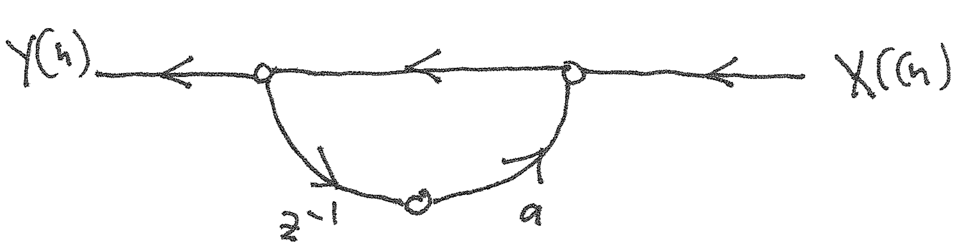
$$H(z) = \frac{1 - \frac{1}{5}z^{-1}}{1 - \frac{1}{4}z^{-1} + \frac{5}{24}z^{-2} + \frac{1}{12}z^{-3}}$$

which is equal to the initial transfer function.

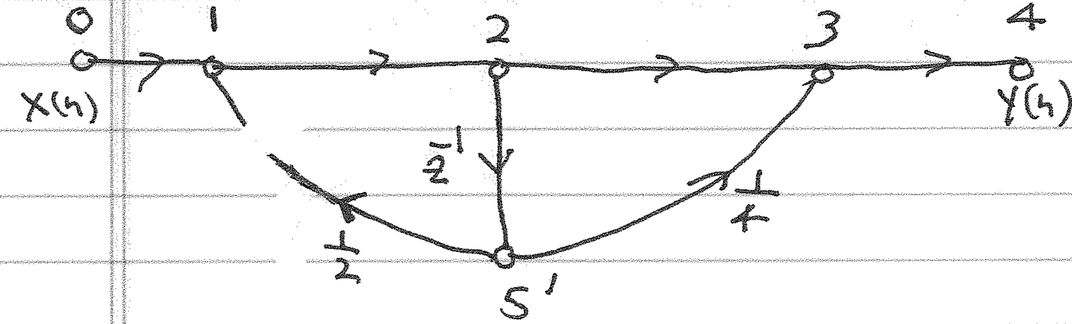
6.24 (a) Original form, with order of computation:



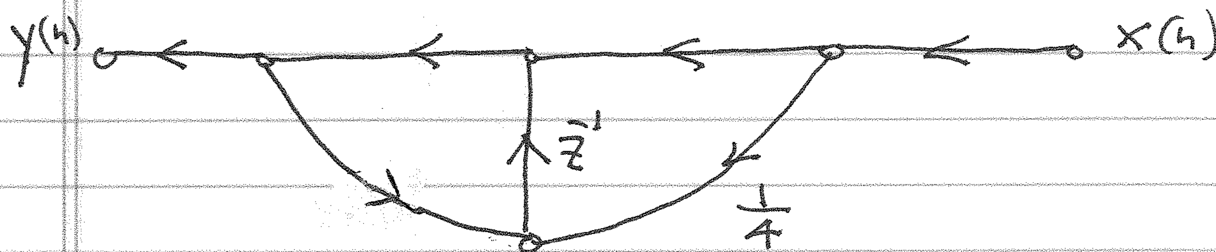
Transpose form:



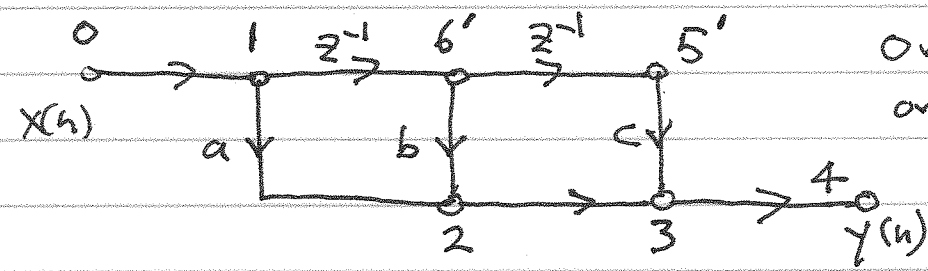
(b) Original form, with order of computation:



Transpose form:

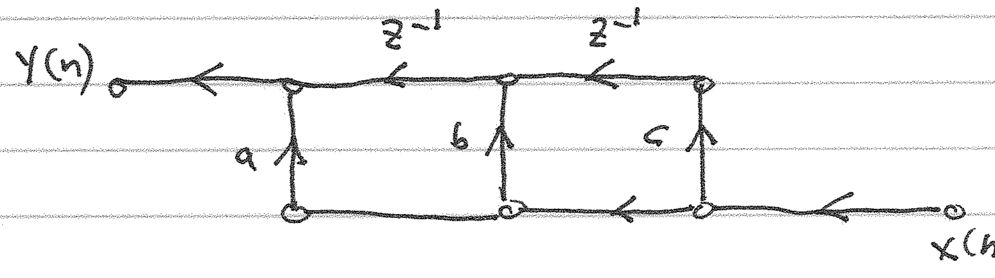


(c)

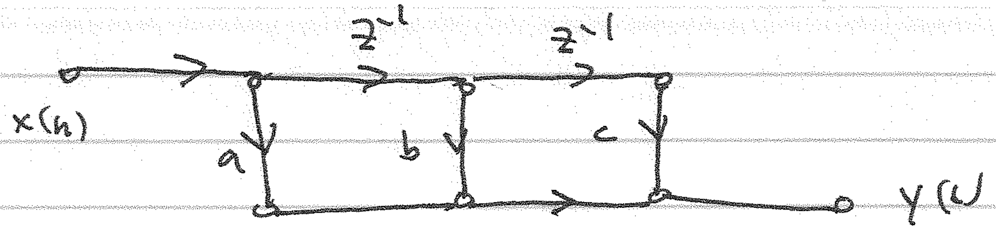


Original form with order of computation

Transpose form:

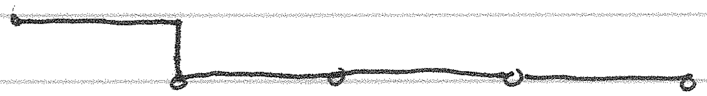


original:



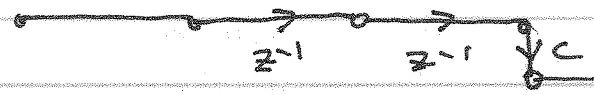
Apply Mason's Rule:

T_1



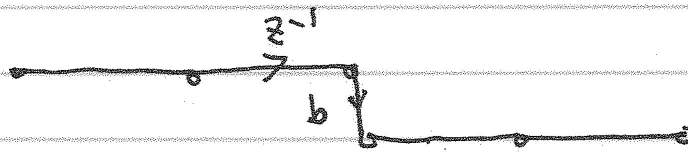
$T_1 = a$

T_2



$T_2 = c z^{-2}$

T_3



$T_3 = b z^{-1}$

(No loops)

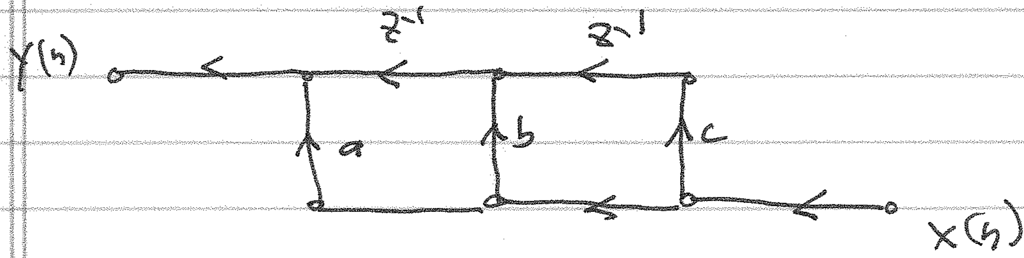
$$T = \frac{\sum T_n \Delta_n}{\Delta}$$

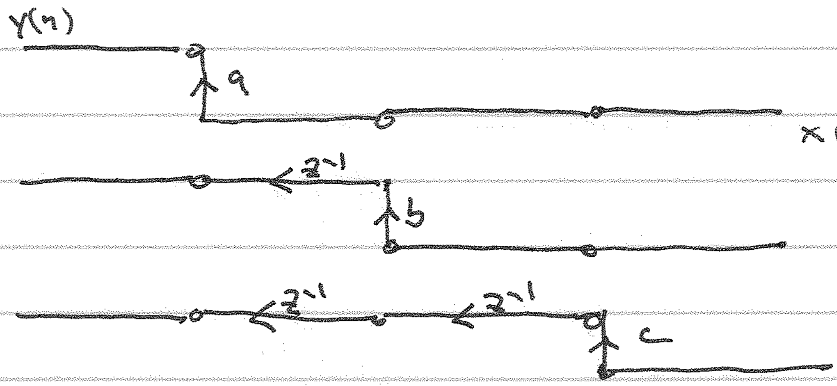
$$\Delta = 1 - \underbrace{\sum L_1 + \sum L_2 - \cdots}_{0} = 1$$

$$\Delta_1 = \Delta_2 = \Delta_3 = 1$$

$$T = T_1 + T_2 + T_3 = a + c z^{-2} + b z^{-1}$$

Transpose form:





$T_1 = a$

$T_2 = b z^{-1}$

$T_3 = c z^{-2}$

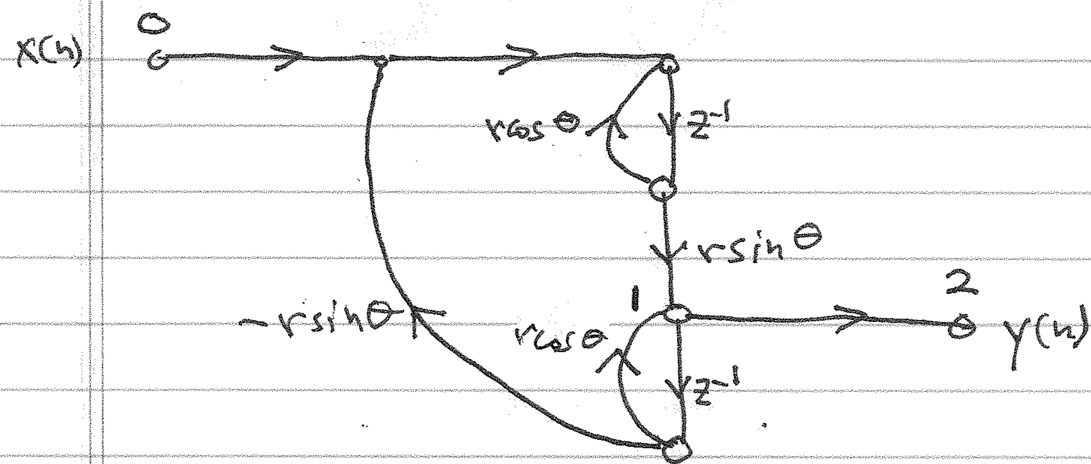
(no loops)

$$T = \frac{\sum T_n \Delta_n}{\Delta} = \frac{a + b z^{-1} + c z^{-2}}{1 - 0}$$

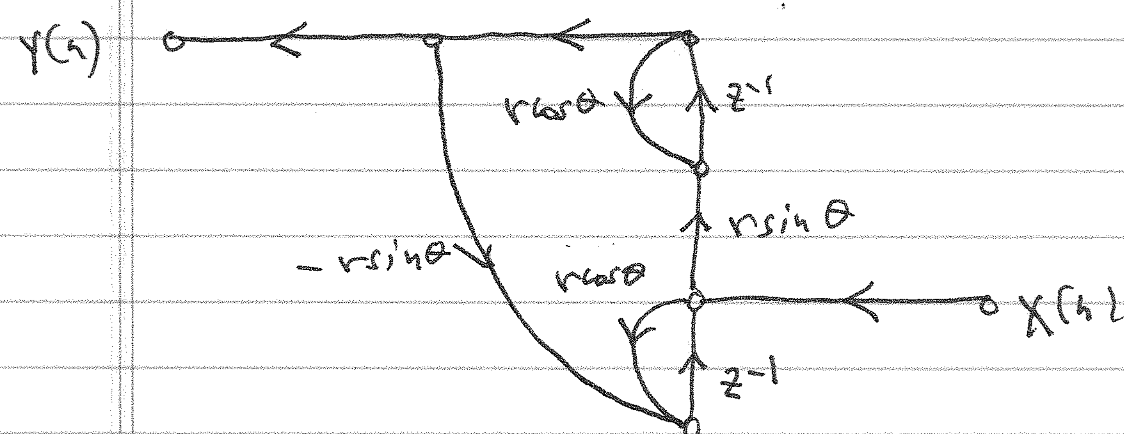
$$= a + b z^{-1} + c z^{-2}$$

(same as T for original form)

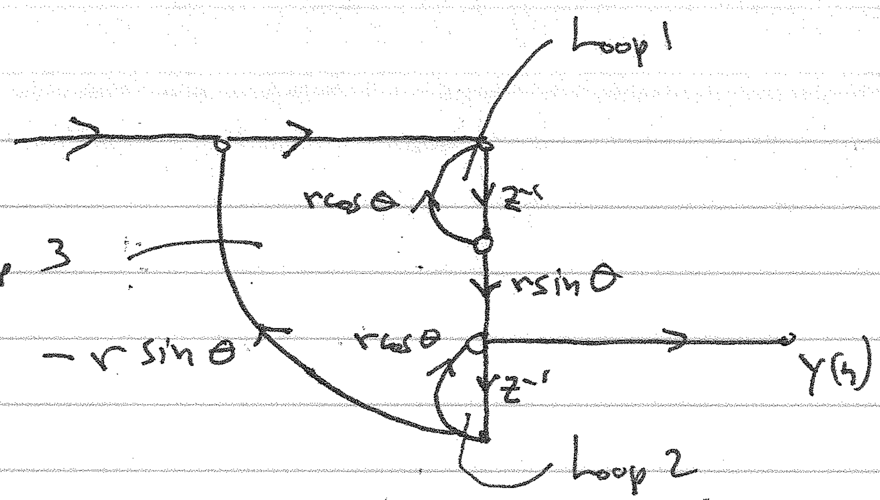
(d) Original form with order of computation



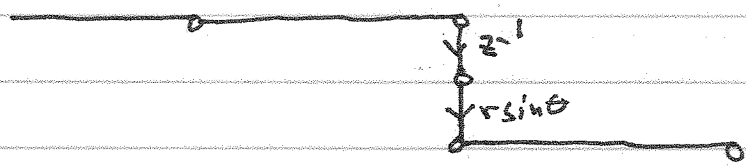
Transpose form:



original:



T_1



$$L1 = r\cos\theta\, z^{-1} \qquad T_1 = r\sin\theta\, z^{-1}$$

$$L2 = r\cos\theta\, z^{-1}$$

$$L3 = -r^2\sin^2\theta\, z^{-2}$$

$$\Delta = 1 - \sum L_1 + \sum L_2$$

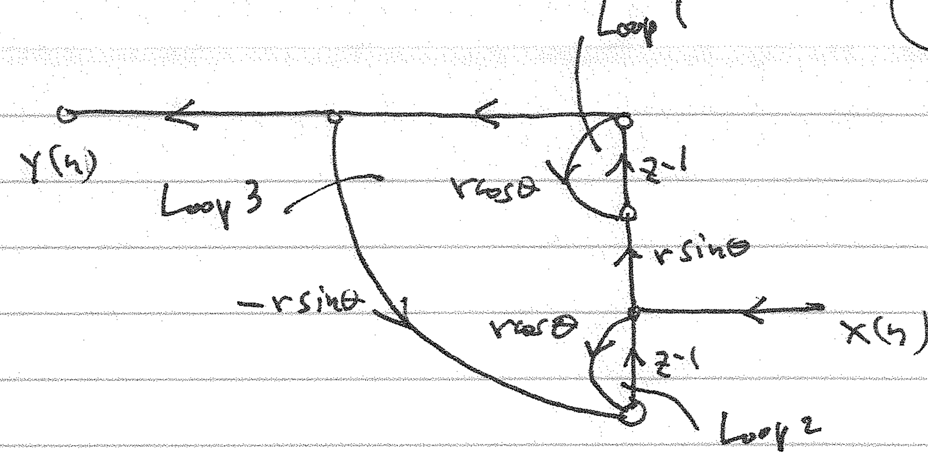
$$= 1 - \left(2r\cos\theta\, z^{-1} - r^2\sin^2\theta\, z^{-2}\right) + r^2\cos^2\theta\, z^{-2}$$

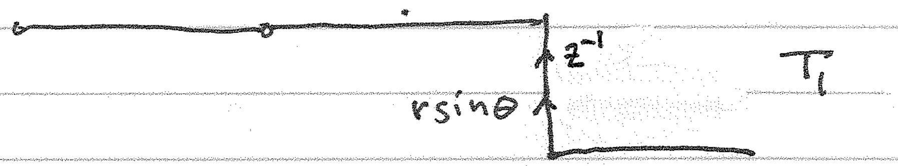
$$= 1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}$$

$\Delta_1 = 1$ Note: Since Loop 2 touches T_1 path, Loop 2 is removed when Δ_1 is calculated.

$$T = \frac{\sum T_n\Delta_n}{\Delta} = \frac{r\sin\theta\, z^{-1}}{1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}}$$

transpose form:





$$T_1 = r\sin\theta\, z^{-1}$$

$$L1 = r\cos\theta\, z^{-1}$$
$$L2 = r\cos\theta\, z^{-1}$$
$$L3 = -r^2\sin^2\theta\, z^{-2}$$

$$\Delta = 1 - \sum L_1 + \sum L_2$$
$$= 1 - (2r\cos\theta\, z^{-1}) + r^2 z^{-2}$$

$$\Delta_1 = 1$$

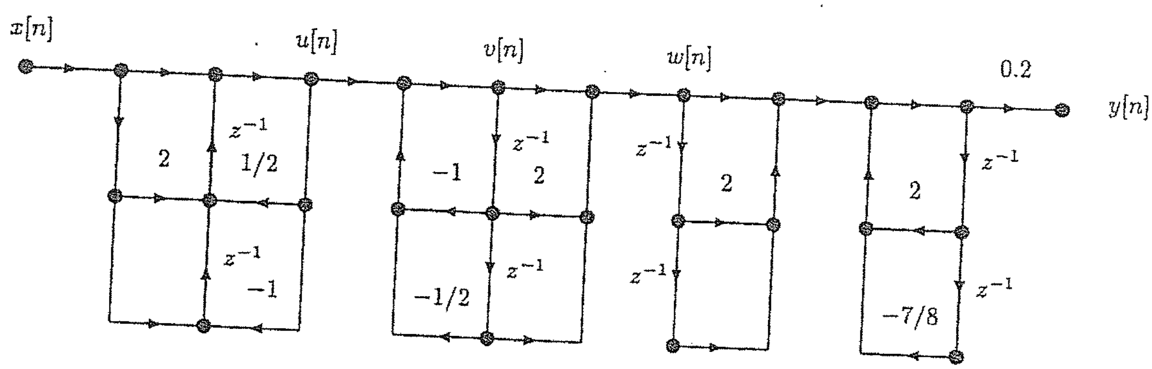
$$T = \frac{\sum T_n \Delta_n}{\Delta} = \frac{r\sin\theta\, z^{-1}}{1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}}$$

(same as T for original form)

11

6.26. (a) We can rearrange $H(z)$ this way:

$$H(z) = \frac{(1+z^{-1})^2}{1-\frac{1}{2}z^{-1}+z^{-2}} \cdot \frac{(1+z^{-1})^2}{1+z^{-1}+\frac{1}{2}z^{-2}} \cdot (1+z^{-1})^2 \cdot \frac{1}{1-2z^{-1}+\frac{7}{8}z^{-2}} \cdot 0.2$$



The solution is not unique; the order of the denominator 2nd-order sections may be rearranged.

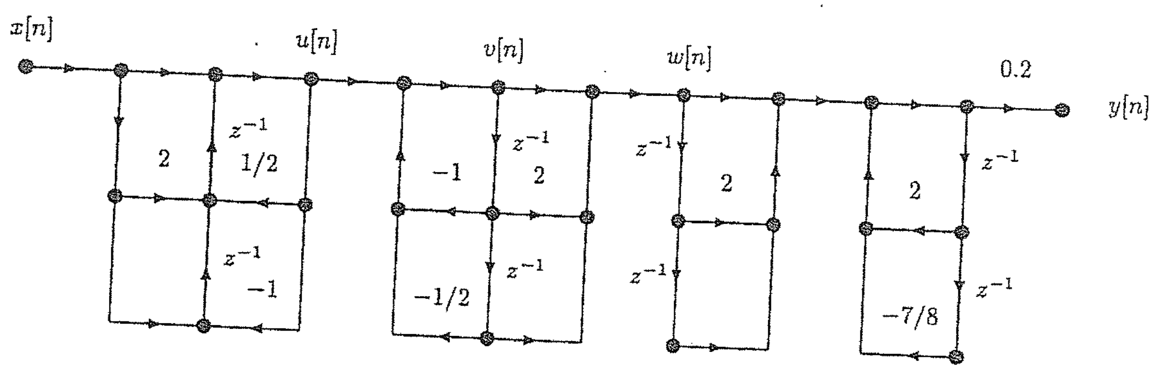
(b)

$$\begin{aligned}
u[n] &= x[n] + 2x[n-1] + x[n-2] + \frac{1}{2}u[n-1] - u[n-2] \\
v[n] &= u[n] - v[n-1] - \frac{1}{2}v[n-2] \\
w[n] &= v[n] + 2v[n-1] + v[n-2] \\
y[n] &= w[n] + 2w[n-1] + w[n-2] + 2y[n-1] - \frac{7}{8}y[n-2].
\end{aligned}$$

11

6.26. (a) We can rearrange $H(z)$ this way:

$$H(z) = \frac{(1+z^{-1})^2}{1-\frac{1}{2}z^{-1}+z^{-2}} \cdot \frac{(1+z^{-1})^2}{1+z^{-1}+\frac{1}{2}z^{-2}} \cdot (1+z^{-1})^2 \cdot \frac{1}{1-2z^{-1}+\frac{7}{8}z^{-2}} \cdot 0.2$$



The solution is not unique; the order of the denominator 2nd-order sections may be rearranged.

(b)

$$\begin{aligned}
u[n] &= x[n] + 2x[n-1] + x[n-2] + \frac{1}{2}u[n-1] - u[n-2] \\
v[n] &= u[n] - v[n-1] - \frac{1}{2}v[n-2] \\
w[n] &= v[n] + 2v[n-1] + v[n-2] \\
y[n] &= w[n] + 2w[n-1] + w[n-2] + 2y[n-1] - \frac{7}{8}y[n-2].
\end{aligned}$$