ECE 844 Homework 7 Solutions

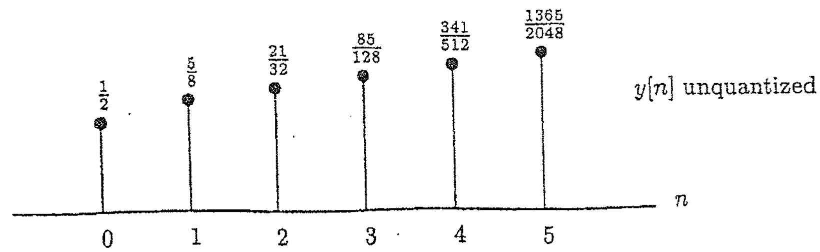
6.43. (a)

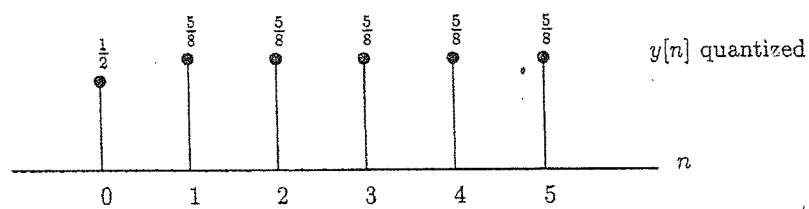
$$y[n] = \frac{1}{4}y[n-1] + \frac{1}{2}, \quad n \geq 0$$

$$y[n] = \frac{1}{2}\sum_{i=0}^{n}\left(\frac{1}{4}\right)^i = \frac{1}{2}\frac{1-\left(\frac{1}{4}\right)^{n+1}}{\frac{3}{4}}$$

For large n, $y[n] = (1/2)/(3/4) = 2/3$.

(b) Working from the difference equation and quantizating after multiplication, it is easy to see that, in the quantized case, $y[0] = 1/2$ and $y[n] = 5/8$ for $n \geq 1$. In the unquantized case, the output monotonically approaches $2/3$.





(c) The system diagram is direct form II:

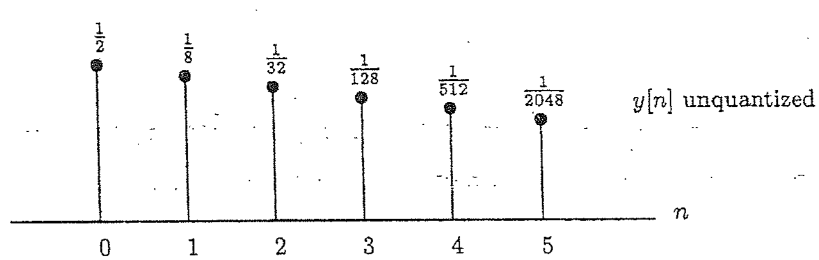
$$H(e^{j\omega}) = \frac{1+e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}}$$

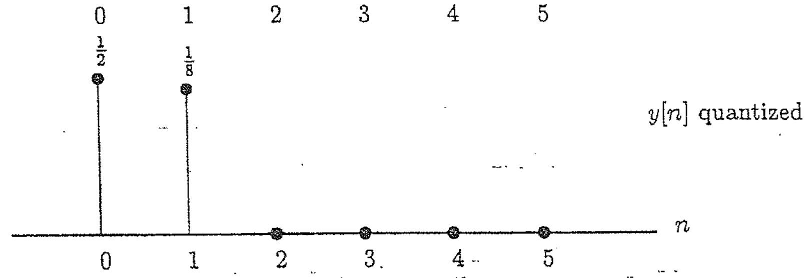
$$X(e^{j\omega}) = \frac{\frac{1}{2}}{1+e^{-j\omega}}$$

So

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}) = \frac{\frac{1}{2}}{1-\frac{1}{4}e^{-j\omega}}$$

which implies that $y[n] = (1/2)(1/4)^n$, which approaches 0 as n grows large.
To find the quantized output (working from the difference equation): $y[0] = 1/2$, $y[1] = 1/8$, and $y[n] = 0$ for $n \geq 2$.





2

6.44. (a) To check for stability, we look at the poles location. The poles are located at

$$z \approx 0.52 + 0.84j \text{ and } z \approx 0.52 - 0.84j.$$

Note that

$$|z|^2 \approx 0.976 < 1.$$

The poles are inside the unit circle, therefore the system function is stable.

(b) If the coefficients are rounded to the nearest tenth, we have

$$1.04 \rightarrow 1.0 \text{ and } 0.98 \rightarrow 1.0.$$

Now the poles are at

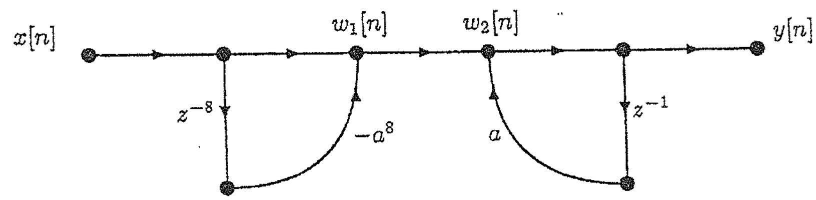
$$z = \frac{1 - j\sqrt{3}}{2} \text{ and } z = \frac{1 - j\sqrt{3}}{2}.$$

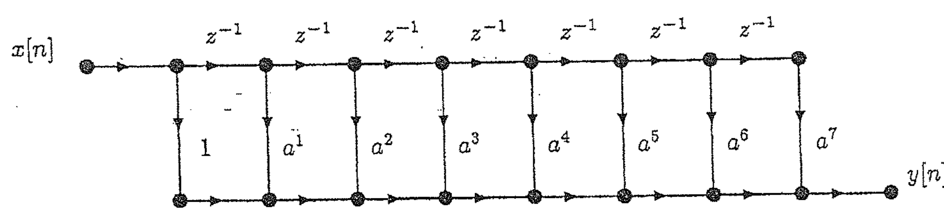
Note that now,

$$|z|^2 = 1.$$

The poles are on the unit circle, therefore the system is not stable.

6.45. The flow graphs for networks 1 and 2 respectively are:





(a) For Network 1, we have:

$$w_1[n] = x[n] - a^8 x[n-8]$$
$$w_2[n] = ay[n-1] + w_1[n]$$
$$y[n] = w_2[n]$$

Taking the Z−transform of the above equations and combining terms, we get:

$$Y(z)(1 - az^{-1}) = (1 - a^8 z^{-8})X(z)$$

That is:

$$H(z) = \frac{1 - a^8 z^{-8}}{1 - az^{-1}}.$$

For Network 2, we have:

$$y[n] = x[n] + ax[n-1] + a^2 x[n-2] + \ldots + a^7 x[n-7].$$

3

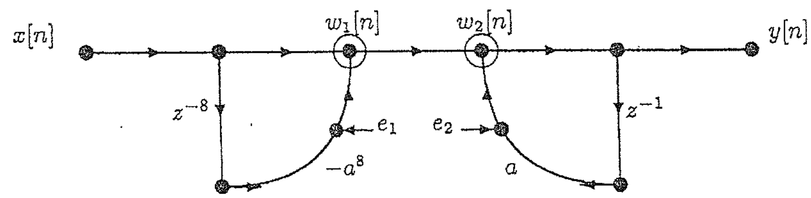
Taking the Z-transform, we get:

$$Y(z) = (1 + az^{-1} + a^2z^{-2} + \ldots + a^7z^{-7})X(z).$$

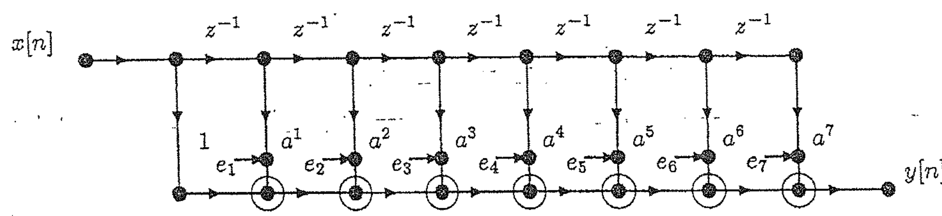
So:

$$H(z) = 1 + az^{-1} + a^2z^{-2} + \ldots + a^7z^{-7} = \frac{1 - a^8z^{-8}}{1 - az^{-1}}.$$

(b) Network 1:



Network 2:



(c) The nodes are circled on the figures in part (b).

(d) In order to avoid overflow in the system, each node in the network must be constrained to have a magnitude less than 1. That is if $w_k[n]$ denotes the value of the kth node variable and $h_k[n]$ denotes the impulse response from the input $x[n]$ to the node variable $w_k[n]$, a sufficient condition for $|w_k[n]| < 1$ is

$$x_{max} < \frac{1}{\sum_{m=-\infty}^{\infty} |h_k[m]|}.$$

In this problem, we need to make sure overflow does not occur in each node, i.e. we need to take the tighter bound on x_{max}. For network 1, the impulse response from $w_2[n]$ to $y[n]$ is $a^n u[n]$, therefore the condition to avoid overflow from that node to the output is

$$w_{max} < 1 - |a|.$$

Where we assumed that $|a| < 1$. The transfer function from $x[n]$ to $w_1[n]$ is $1 - a^8z^{-8}$, therefore to avoid overflow at that node we need:

$$w_1[n] < x_{max}(1 - a^8) < 1 - |a|.$$

We thus conclude that to avoid overflow in network 1, we need:

$$x_{max} < \frac{1-|a|}{1-a^8}.$$

Now, for network 2, the transfer function from input to output is given by $\delta[n] + a\delta[n-1] + a^2\delta[n-2] + ... + a^7\delta[n-7]$, therefore to avoid overflow, we need:

$$x_{max} < \frac{1}{1+|a|+a^2+...+|a|^7}.$$

(e) For network 1, the total noise power is $\frac{2\sigma_e^2}{1-|a|^2}$ For network 2, the total noise power is $7\sigma_e^2$. For network 1 to have less noise power than network 2, we need

$$\frac{2\sigma_e^2}{1-|a|^2} < 7\sigma_e^2.$$

That is:

$$|a|^2 < \frac{5}{7}. \quad \Rightarrow |a| < \sqrt{\frac{5}{7}}$$

The largest value of $|a|$ such that the noise in network 1 is less than network 2 is therefore $\sqrt{\frac{5}{7}}$.