Solutions

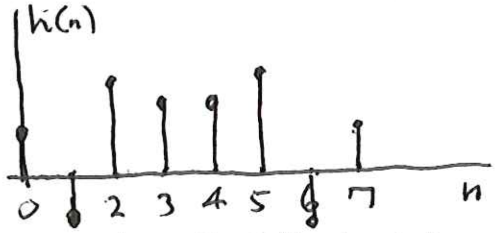
ECE 8440 Quiz 2 March 31, 2015

---

100 Points 6 problems 90 minutes 2 pages of notes permitted

---

1. (16 points) Consider a generalized Type II linear phase FIR digital filter whose unit sample response h(n) extends from n = 0 through n = 7. Provide a feasible plot of h(n) for this filter.



(b) You are given the following information about some of the zeros of the system function H(z) of this filter:

- One zero is located at $z = .8\, e^{j\pi/8}$

- One zero is located at z = .5

Based on the above information, give as much information as you can regarding the location of the other zeros of H(z).

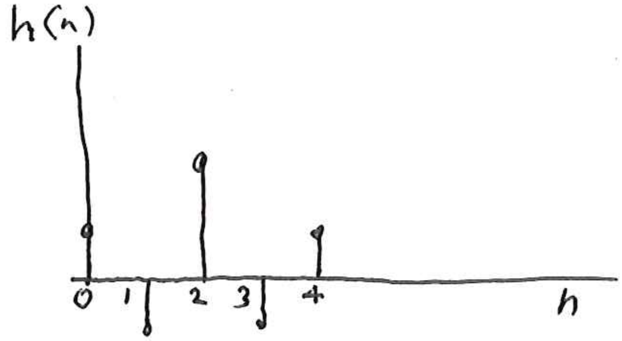
Other zeros of H(z) are at:

$z = .8 e^{-j\pi/8}, \left(\frac{1}{.8}\right)e^{j\pi/8}, \left(\frac{1}{.8}\right)e^{-j\pi/8}$ (to complete a group of 4)

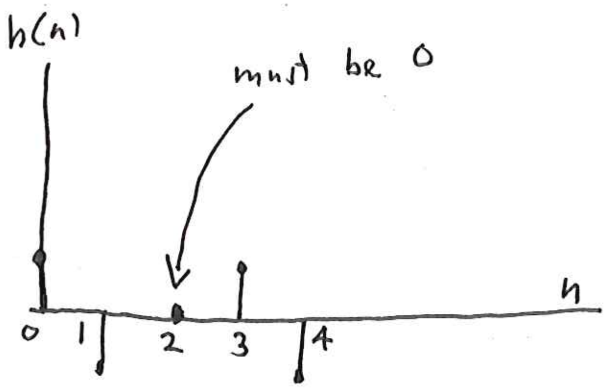
$z = 2$ (to complete a group of 2 real zeros)

$z = -1$ (since a Type II filter always has a zero at $z = -1$)

2. (16 points) (a) Give a plot of a feasible unit sample response h(n) for a causal Type I FIR digital filter which has a duration of at least 5.



(b) Give a plot of a feasible unit sample response h(n) for a causal Type III FIR digital filter which has a duration of at least 5.



(c) Which Type(s) of FIR digital filters <u>cannot</u> be used for low pass filters?

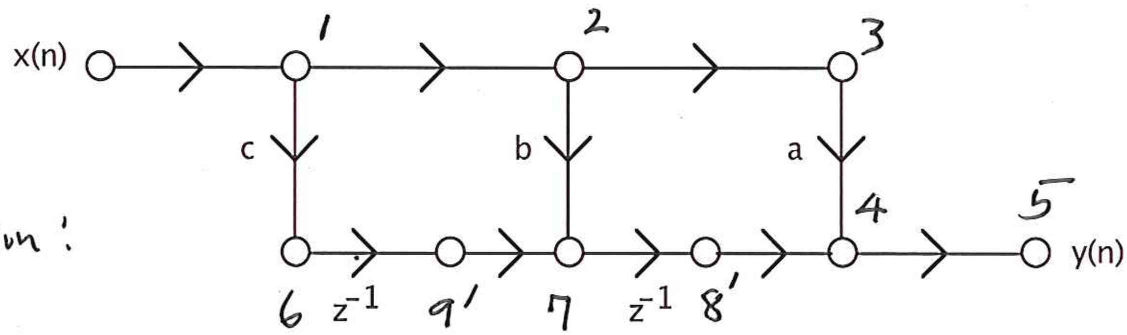
Type III and Type IV, since they always have a zero at $z = 1$ $(\omega = 0)$.

(d) Which Type(s) of FIR digital filters <u>cannot</u> be used for high pass filters?

Type II and Type III, since they always have a zero at $z = -1$ $(\omega = \pi)$.

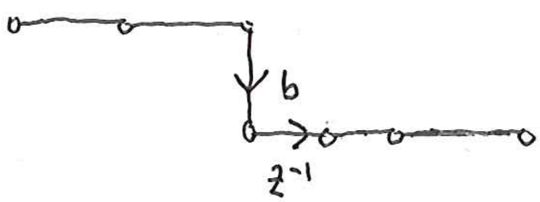
3. (16 points) (a) In the figure below, indicate the order in which the nodes should be updated each time a new input appears at x(n). Let "Node 0" be the name of the node where x(n) is applied.

one solution !

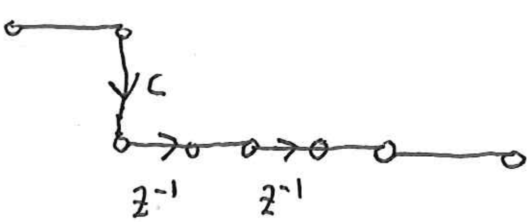


(b) Use Mason's Rule to obtain the system function H(z) for the above system.

$T_1 = a$



$T_2 = bz^{-1}$



$T_2 = cz^{-2}$

$$H(z) = \frac{\sum_{n=1}^{3} T_n \Delta_n}{\Delta}$$

where $\Delta = 1 - \underbrace{\Sigma L_1}_{0} + \underbrace{\Sigma L_2}_{0} - \underbrace{\Sigma L_3}_{0} \ldots$

$= 1$

$\Rightarrow \boxed{H(z) = a + bz^{-1} + cz^{-2}}$

and $\Delta_1 = \Delta_2 = \Delta_3 = 1$

4. (16 points) Consider a 8-th order IIR digital filter which is to be implemented as a cascade of four 2nd order sections. (Each section includes two complex conjugate zeros and two complex conjugate poles.) Assume that any of these sections can be implemented as Direct Form I or as Direct Form II. (All sections don't have to be implemented using the same Form.)

What is the total number of possible implementation configurations, based on the above information? (Consider all possible ways of pairing poles with zeros, ordering the 2nd order sections, and selecting the filter Forms listed above.)

pairing options : $4!$

ordering options : $4!$

form combination options : 2^4

$\Rightarrow$ Total no. of possible implementation configurations, based on information given in the problem is

$$(4!)(4!)(2^4) = \boxed{9216}$$

5. (16 points) Consider the following 2-nd order digital filter which has two complex conjugate poles and two complex conjugate zeros.

$$H(z) = \frac{.2 - .5z^{-1} + .1z^{-2}}{1 - .9z^{-1} + .81z^{-2}}$$

If this filter is implemented using Direct Form I, determine the power of the noise in the output due to internal round-off noise. Assume that all data must be represented using a 8-bit 2's complement fixed-point format that can represent data values between -1 and 1. Also assume that double-length accumulators and registers are not available. (Hint: You can answer this problem by appropriately applying the formula developed in class, which was based on a z-transform approach described in an appendix of the text.

Since $H(z)$ is to be implemented using Direct Form I, the 5 round-off errors pass only through the poles of $H(z)$. Therefore the power of noise in the output due to internal round-off error is:

$$5\sigma_e^2\left(\frac{1+r^2}{1-r^2}\right)\frac{1}{1-2r^2\cos(2\theta)+r^4}$$, where the poles of $H(z)$ are at $re^{\pm j\theta}$

and where $\sigma_e^2 = \left.\frac{2^{-2B}}{12}\right|_{B=7}$

Find r and θ:

$$1 - .9z^{-1} + .81z^{-2} = (1 - re^{j\theta}z^{-1})(1 - re^{-j\theta}z^{-1})$$
$$= 1 - 2r(\cos\theta)z^{-1} + r^2z^{-2} \quad \Rightarrow r = .9$$

and $-2(.9)\cos\theta = -.9 \quad \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} \Rightarrow 2\theta = \frac{2\pi}{3}$

$\cos(2\theta) = -\frac{1}{2}$

$\Rightarrow$ power of noise in output is:

$$5\underbrace{\left(\frac{2^{-14}}{12}\right)}_{5.0863\times10^{-6}}\underbrace{\frac{(1+.81)}{(1-.81)}}_{9.5263}\underbrace{\frac{1}{1-2(.81)(-\frac{1}{2})+(.81)^2}}_{.4055} = \boxed{9.8239 \times 10^{-5}}$$

6. (20 points) Consider a 4th order digital filter implemented as a cascade of two 2[nd] order sections.

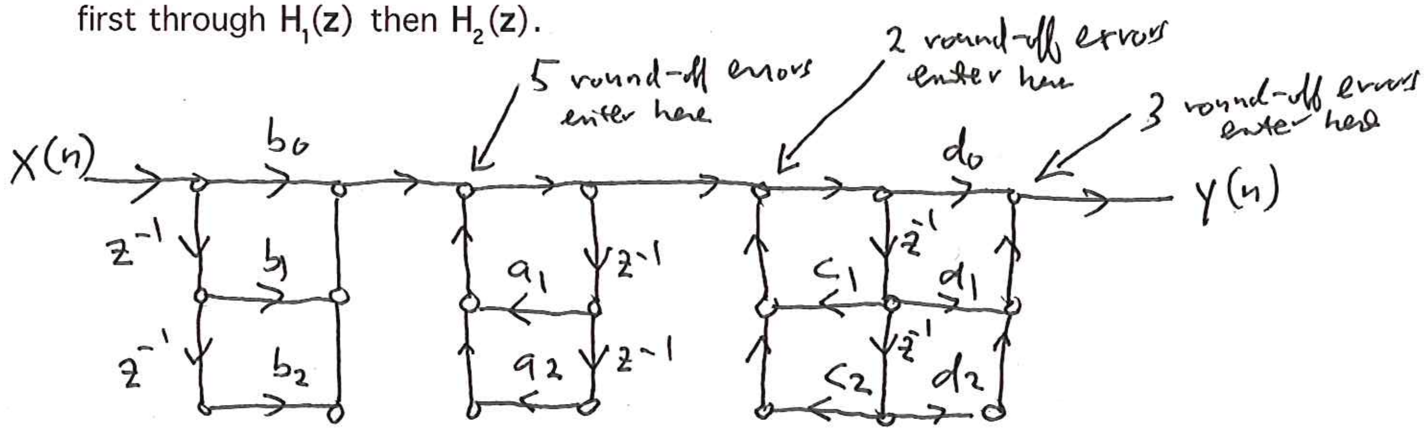
$H(z) = H_1(z)H_2(z)$ where

$$H_1(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}} \quad \text{and}$$

$$H_2(z) = \frac{D(z)}{C(z)} = \frac{d_0 + d_1 z^{-1} + \cancel{b_2}\, d_2 z^{-2}}{1 - c_1 z^{-1} - c_2 z^{-2}}$$

Assume that both $H_1(z)$ and $H_2(z)$ is implemented using Direct Form I and $H_2(z)$ is implemented using Direct Form II.

Provide a signal flow graph for $H(z)$, assuming that the input signal passes first through $H_1(z)$ then $H_2(z)$.



(b) For the filter implementation described in part 'a', give an expression for the total average power in the system output due to round-off noise. Give your answer in terms of mathematical expression involving σ_e^2, the expected squared value of an individual round-off error source, and an appropriate subset of the following: $H_1(e^{j\omega})$, $H_2(e^{j\omega})$, $A(e^{j\omega})$, $B(e^{j\omega})$, $C(e^{j\omega})$, and $D(e^{j\omega})$.

(Assume that double length registers and accumulators are NOT available.)

Total average power in the system output due to round-off noise is : (See figure on previous page)

$$5\sigma_e^2 \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{|A(e^{j\omega})|^2} \cdot |H_2(e^{j\omega})|^2 d\omega + 2\sigma_e^2 \frac{1}{2\pi}\int_{-\pi}^{\pi} |H_2(e^{j\omega})|^2 d\omega + 3\sigma_e^2$$