ECE 8440 Review for Quiz 2 **Spring 2015**

Units 9,10 - Generalized Linear Phase

$H(e^{j\omega}) = A(e^{j\omega})e^{-j\alpha\omega+\beta}$ (equation 5.125)

where $A(e^{j\omega})$ is real, but may be negative (and therefore contribute π to the phase).

For a system having generalized linear phase,

$$\sum_{n=-\infty}^{\infty} h(n)\left[\sin[(\beta + \omega(n-\alpha))]\right] = 0$$ (equation 5.130)

If $\beta = 0$ or $\beta = \pi$ and if $2\alpha = \text{integer}$,

then

$h(n) = h(2\alpha - n)$, all n (equation 5.131c)

This is the same as

$h(\alpha + n) = h(\alpha - n)$

If $\beta = \pi / 2$ or $\beta = 3\pi / 2$ and if $2\alpha = \text{integer}$,

then

$h[2\alpha - n] = -h(n)$. (equation 5.133c)

This is the same as

$h(\alpha + n) = -h(\alpha - n)$

Note: If $2\alpha \neq \text{integer}$, then neither of the above symmetry conditions hold true.

Causal version of above symmetry conditions when $2\alpha = \text{integer}$:

$h(n) = h(M-n), \quad 0 \le n \le M$ (Types I and II filters) and $h(n) = -h(M-n), \quad 0 \le n \le M$ (Types III and IV filters).

Location of zeros for Types I-IV generalized linear phase filters

If there is a zero at $z_0 = r_0 e^{j\omega_0}$, then there must also be a zero at $\frac{1}{z_0} = \frac{1}{r_0} e^{-j\omega_0}$

If order for h(n) to be real, the complex conjugates of each of the above zeros must also be included in H(z). This implies a group of 4 zeros:

$$r_0 e^{j\omega_0} \quad , \quad r_0 e^{-j\omega_0} \quad , \quad \frac{1}{r_0} e^{j\omega_0} \quad , \text{ and } \quad \frac{1}{r_0} e^{-j\omega_0}$$

Special cases:

If $r = 1$ and $\theta \ne 0, \pi$: Group size = 2: $e^{j\omega_0}$ and $e^{-j\omega_0}$

If $r \ne 1$ and $\theta = 0$ or π: Group size = 2: r and $\frac{1}{r}$ or $-r$ and $-\frac{1}{r}$

If $r = 1$ and $\theta = 0$ or π: Group size = 1: 1 or -1

Unit 11 (Filter Structures)

- Direct Form I
- Direct Form II

-Cascade Forms
-Parallel Forms

- Transpose Forms

-Signal flow graphs
    - Find H(z) using node equations in signal flow graph
    - Find H(z) using Mason's Rule
    - Determine order of node updating in signal flow graphs.

- FIR structures that take advantage of symmetry of h(n) for Types I-IV filters.

Unit 12 (Using Finite Precision Arithmetic to Implement Filters)

Two's Complement Representation Using B+1 bits

If a number is represented as a two's complement fraction having the form

$$x = X_m\left(-b_0 + \sum_{i=1}^{\infty} b_i 2^{-i}\right) \qquad \text{(equation 6.75)}$$

Then the resolution for representing the value is

$$\Delta = X_m 2^{-B}$$

and the quantization error will be bounded by

$$-(\Delta / 2) < e \leq (\Delta / 2).$$

Coefficient quantization

If filter coefficients are quantized, the effect is:

- finite number of possible location of poles a zeros
- finite number of possible frequency response functions

Unit 13 - Round-off Noise due to Multiplications in Digital Filters

Assume that the input e(n) is white noise due to round-off and the average power of this noise is σ_e^2. Also assume that the frequency response of that portion of the system between the noise entry point and the system output f(n) is $H_{ef}(e^{j\omega})$.

The power density spectrum of the noise in the output is due to round-off error with average power of σ_e^2 is

$$\Phi_{ff}(e^{j\omega}) = \sigma_e^2 \, | H_{ef}(e^{j\omega}) |^2 \qquad \text{(equation 6.103)}$$

where $H_{ef}(e^{j\omega})$ is the frequency response of that portion of the system between the noise entry point and the system output.

The average power of the output noise due to round-off error is

$E[f^2(n)]$ which is equal to $\varphi_{yy}(0)$

where

$$\varphi_{ff}(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_{ff}(e^{j\omega}) d\omega$$

$$= \frac{\sigma_e^2}{2\pi} \int_{-\pi}^{\pi} | H_{ef}(e^{j\omega}) |^2 d\omega$$

Applying Parseval's relation, we can also express the above as

$$\varphi_{ff}(0) = \sigma_e^2 \sum_{\ell=-\infty}^{\infty} | h_{ef}(k) |^2$$

Using a z-transform approach to finding $\varphi_{ff}(0)$:

$$\Gamma_{ff}(z) = \sigma_e^2 H_{ef}(z) H_{ef}^{*}\left(\frac{1}{z^{*}}\right)$$

$$\sigma_f^2 = \gamma_{yy}(0) = \sigma_e^2\left(\sum_{k=1}^{N} A_k\right)$$

where

$$\Gamma_{ff}(z) = \sigma_e^2\left(\sum_{k=1}^{N}\frac{A_k}{1-d_k z^{-1}} - \frac{A_k^{*}}{1-(d_k^{*})^{-1}z^{-1}}\right) \qquad \text{(equation A.64)}$$

and where

$$H(z) = A\frac{\prod_{m=1}^{M}(1-c_m z^{-1})}{\prod_{k=1}^{N}(1-d_k z^{-1})}$$

Example: For the following second order filter with no zeros,

$$H(z) = \frac{1}{(1-re^{j\theta}z^{-1})(1-re^{-j\theta}z^{-1})}$$

$$A_1 + A_2 = \left(\frac{1+r^2}{1-r^2}\right)\frac{1}{1-2r^2\cos(2\theta)+r^4}$$

so that

$$\sigma_f^2 = \gamma_{yy}(0) = \sigma_e^2\left(\frac{1+r^2}{1-r^2}\right)\frac{1}{1-2r^2\cos(2\theta)+r^4}$$

Unit 14 - More on Round-Off Noise in Digital Filters

If products due to multiplication are rounded to a B+1 bit two-complement representation, the average noise power of each round-off noise source is

$$\sigma_e^2 = \frac{2^{-2B}}{12}$$

- Points of injection of round-off noise for different filter structures

- Combining round-off noise sources to a larger equivalent noise source in different filter structures

For Direct Form I

Equivalent noise source:

$$\sigma_e^2 = (M+1+N)\frac{2^{-2B}}{12}$$

and

$$H_{ef}(z) = \frac{1}{A(z)}$$

where

$$H(z) = \frac{B(z)}{A(z)}.$$

The total output noise power due to internal round-off error is therefore

$$\sigma_f^2 = (M+1+N)\frac{2^{-2B}}{12}\int_{-\pi}^{\pi}\frac{d\omega}{|A(e^{j\omega})|^2} \qquad \text{(equation 6.106)}$$

$$= (M+1+N)\frac{2^{-2B}}{12}\sum_{n=-\infty}^{\infty}|h_{ef}(n)|^2,$$

where $h_{ef}(n)$ is the unit sample response corresponding to $H_{ef}(z) = \frac{1}{A(z)}$.

Note: The z-transform method for finding $\sigma_f^2 = \gamma_{yy}(0)$ could also be used.

For Direct Form II

The total average noise power in the output can be expressed as

$$\sigma_f^2 = N\frac{2^{-2B}}{12}\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H(e^{j\omega})\right|^2 d\omega + (M+1)\frac{2^{-2B}}{12}$$

or as

$$\sigma_f^2 = N\frac{2^{-2B}}{12}\sum_{n=-\infty}^{\infty}\left|h[n]\right|^2 + (M+1)\frac{2^{-2B}}{12}$$

A third option is to use the z-transform based approach to find σ_f^2 as

$$\sigma_f^2 = \gamma_{ff}(0).$$

Scaling to Prevent Overflow

Assume that an input signal is bounded by

$$| x(n) \leq x_{max}$$

To guarantee no overflow at all critical nodes of a filter, multiply the input x(n) by a scale factor s where the value of **s** is selected to satisfy:

$$s \leq \frac{1}{(x_{max})\max_k\left[\sum_{m=\infty}^{\infty} | h_k(m) |\right]}$$

where

$h_k(m)$ is the unit sample response of the part of the system between the input and node k.

A less conservative method to choosing s: Choose s to satisfy

$$s \leq \frac{1}{(x_{max})\max_{k,|\omega|<\pi} |H_k(e^{j\omega})|}$$

A third (and even less conservative) method for choosing s is

$$s \leq \frac{1}{\sqrt{\left(\sum_{n=\infty}^{\infty} | x(n) |^2\right) \max_k \sum_{n=\infty}^{\infty} | h_k(n) |^2}}.$$

- Critical nodes for scaling when non-saturating two-complement arithmetic is used

    - For Direct Form I (output node only)   )   (see figure 6.59)
    - For Direct Form II (input node and output node)   (see figure 6.61)

Unit 15 - More on Scaling and Round-Off Noise

Interaction between scaling and round-off noise

-Reducing the input magnitude by scaling can reduce the ratio of signal-to-quantiation noise in the output. (Review example 6.13)

- It is best to use distributed scaling in cascade implementations instead of reducing the entire signal as it enters filter (and reducing none of the internally generated round off noise)

- Procedure for implementing distributed scaling (see example in section 6.9.3)

Grouping of poles and zeros and ordering of second order sections in cascade implementations of digital filters

- No. of possible ways to group N complex conjugate zero pairs with N complex conjugate pole-pairs is $N!$

- There are $N!$ ways to order the resulting N second order sections.

- If we consider four options for implementing each second order section (e.g., Direct Form I, Direct Form II, transpose of Direct Form I, and transpose of Direct Form II), the total number of distinct implementation options is $4(N!)^2$

"Rules of thumb" for selecting the desired configuration of second order section for a filter:

1. Pair the pole that is closest to the unit circle with the zero that is closest to it.

2. Repeat Rule 1 until all poles and zeros have been paired.

3. Order the resulting second-order sections according to either increasing closeness to the unit circle OR decreasing closeness to the unit circle.

Limit Cycles

- What a limit cycle is
- What causes a limit cycle