ECE 8440 Spring 2017
Homework 1: Due February 6

From the following problems (of the 2nd edition of the text):

4.22 (Hint for part b: Up-sample x(n) by a factor of 2; then apply the appropriate digital filter followed by an ideal D/C converter that operates at the appropriate conversion rate).

4.24, 4.27, 4.29, 4.32, 4.41

4.22. A continuous-time signal $x_c(t)$, with Fourier transform $X_c(j\Omega)$ shown in Figure P4.22-1, is sampled with sampling period $T = 2\pi/\Omega_0$ to form the sequence $x[n] = x_c(nT)$.

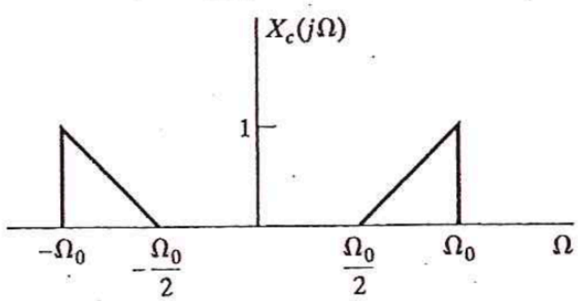

Figure P4.22-1

(a) Sketch the Fourier transform $X(e^{j\omega})$ for $|\omega| < \pi$.

(b) The signal $x[n]$ is to be transmitted across a digital channel. At the receiver, the original signal $x_c(t)$ must be recovered. Draw a block diagram of the recovery system and specify its characteristics. Assume that ideal filters are available.

(c) In terms of Ω_0, for what range of values of T can $x_c(t)$ be recovered from $x[n]$?

4.24. In the system of Figure P4.24-1, $X_c(j\Omega)$ and $H(e^{j\omega})$ are as shown. Sketch and label the Fourier transform of $y_c(t)$ for each of the following cases:

(a) $1/T_1 = 1/T_2 = 10^4$

(b) $1/T_1 = 1/T_2 = 2 \times 10^4$

(c) $1/T_1 = 2 \times 10^4, \quad 1/T_2 = 10^4$

(d) $1/T_1 = 10^4, \quad 1/T_2 = 2 \times 10^4$

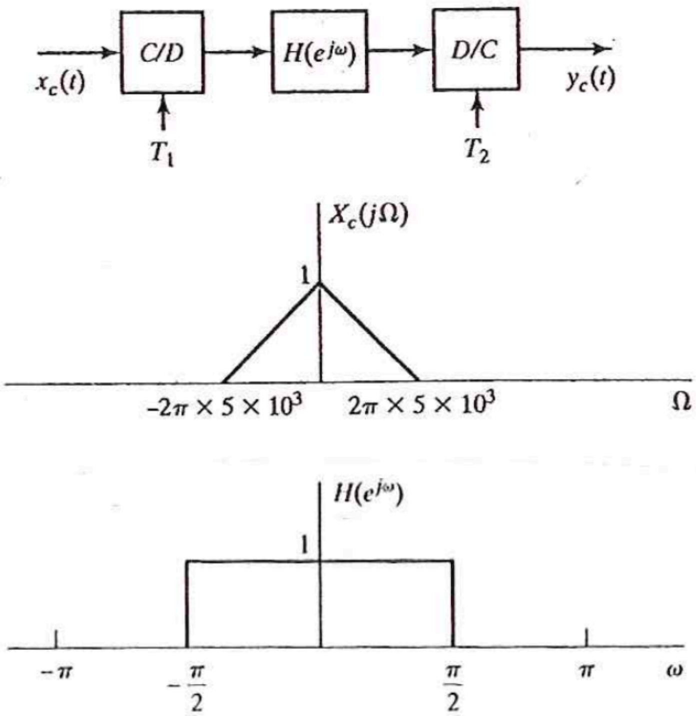

Figure P4.24-1

4.27. Using Parseval's theorem, briefly explain why the amplitude of the Fourier transform changes during downsampling but not during upsampling.

4.29. Consider the systems shown in Figure P4.29-1. Suppose that $H_1(e^{j\omega})$ is fixed and known. Find $H_2(e^{j\omega})$, the frequency response of an LTI system, such that $y_2[n] = y_1[n]$ if the inputs to the systems are the same.

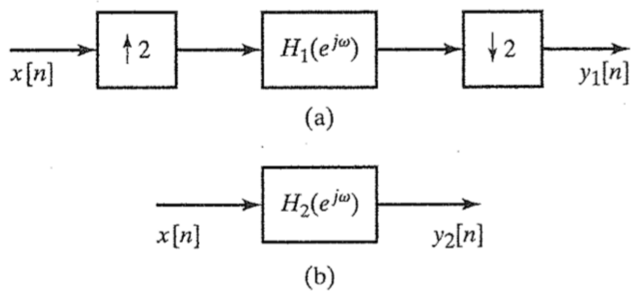

Figure P4.29-1

4.32. A bandlimited continuous-time signal is known to contain a 60-Hz component, which we want to remove by processing with the system of Figure 4.11, where $T = 10^{-4}$.

(a) What is the highest frequency that the continuous-time signal can contain if aliasing is to be avoided?

(b) The discrete-time system to be used has frequency response

$$H(e^{j\omega}) = \frac{[1 - e^{-j(\omega-\omega_0)}][1 - e^{-j(\omega+\omega_0)}]}{[1 - 0.9e^{-j(\omega-\omega_0)}][1 - 0.9e^{-j(\omega+\omega_0)}]}.$$

Sketch the magnitude and phase of $H(e^{j\omega})$.

(c) What value should be chosen for ω_0 to eliminate the 60-Hz component?

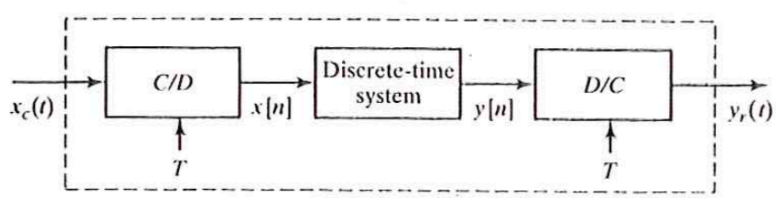

Figure 4.11 Discrete-time processing of continuous-time signals.

4.41. Consider the system shown in Figure P4.41-1. The input to this system is the bandlimited signal whose Fourier transform is shown in Figure P4.20-1 with $\Omega_0 = \pi/T$. The discrete-time LTI system in Figure P4.41-1 has the frequency response shown in Figure P4.41-2.

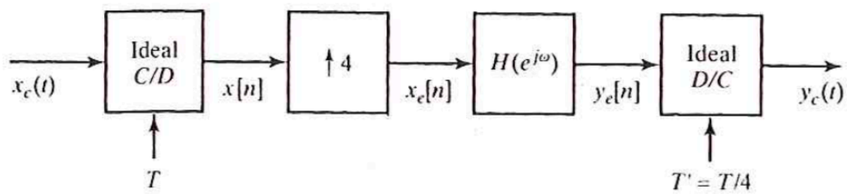

Figure P4.41-1

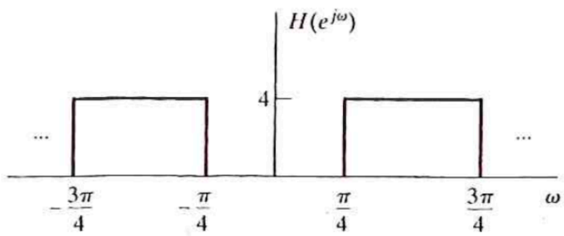

Figure P4.41-2

(a) Sketch the Fourier transforms $X(e^{j\omega})$, $X_e(e^{j\omega})$, $Y_e(e^{j\omega})$, and $Y_c(j\Omega)$.

(b) For the general case when $X_c(j\Omega) = 0$ for $|\Omega| \geq \pi/T$, express $Y_c(j\Omega)$ in terms of $X_c(j\Omega)$. Also, give a general expression for $y_c(t)$ in terms of $x_c(t)$ when $x_c(t)$ is bandlimited in this manner.

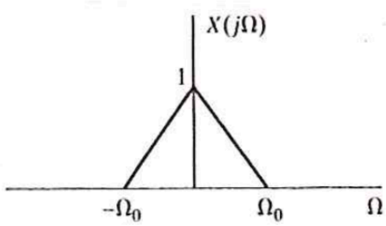

Figure P4.20-1