ECE 8440 Spring 2017
Homework 4: Due Wed, Mar 15

6.43, 6.44, 6.45

6.43. The flow graph of a first-order system is shown in Figure P6.43-1.

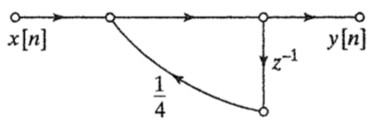


Figure P6.43-1

(a) Assuming infinite-precision arithmetic, find the response of the system to the input

$$x[n] = \begin{cases} \frac{1}{2}, & n \geq 0, \\ 0, & n < 0. \end{cases}$$

What is the response of the system for large n?

Now suppose that the system is implemented with fixed-point arithmetic. The coefficient and all variables in the network are represented in sign-and-magnitude notation with 5-bit registers. That is, all numbers are to be considered signed fractions represented as:

$$b_0 b_1 b_2 b_3 b_4,$$

where b_0, b_1, b_2, b_3, and b_4 are either 0 or 1 and

$$|\text{Register value}| = b_1 2^{-1} + b_2 2^{-2} + b_3 2^{-3} + b_4 2^{-4}.$$

If $b_0 = 0$, the fraction is positive, and if $b_0 = 1$, the fraction is negative. The result of a multiplication of a sequence value by a coefficient is truncated before additions occur; i.e., only the sign bit and the most significant four bits are retained.

(b) Compute the response of the quantized system to the input of Part (a), and plot the responses of both the quantized and unquantized systems for $0 \leq n \leq 5$. How do the responses compare for large n?

(c) Now consider the system depicted in Figure P6.43-2, where

$$x[n] = \begin{cases} \frac{1}{2}(-1)^n, & n \geq 0, \\ 0, & n < 0. \end{cases}$$

Repeat Parts (a) and (b) for this system and input.

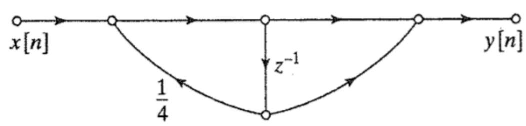


Figure P6.43-2

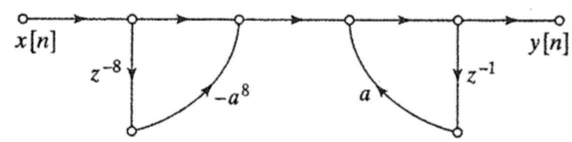
6.44. A causal LTI system has a system function

$$H(z) = \frac{1}{1 - 1.04z^{-1} + 0.98z^{-2}}.$$

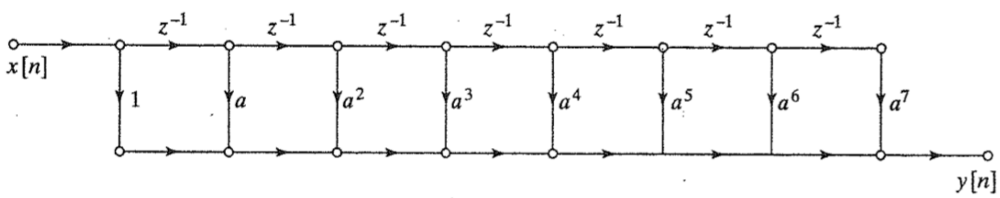
(a) Is this system stable?

(b) If the coefficients are rounded to the nearest tenth, would the resulting system be stable?

6.45. When implemented with infinite-precision arithmetic, the flow graphs in Figure P6.45-1 have the same system function.



Network 1



Network 2

Figure P6.45-1

(a) Show that the two systems have the same overall system function from input $x[n]$ to output $y[n]$.

(b) Assume that the preceding systems are implemented with two's complement fixed-point arithmetic and that products are rounded *before* additions are performed. Draw signal flow graphs that insert round-off noise sources at appropriate locations in the signal flow graphs of Figure P6.45-1.

(c) Circle the nodes in your figure from Part (b) where overflow can occur.

(d) Determine the maximum size of the input samples such that overflow cannot occur in either of the two systems.

(e) Assume that $|a| < 1$. Find the total noise power at the output of each system, and determine the maximum value of $|a|$ such that Network 1 has lower output noise power than Network 2.