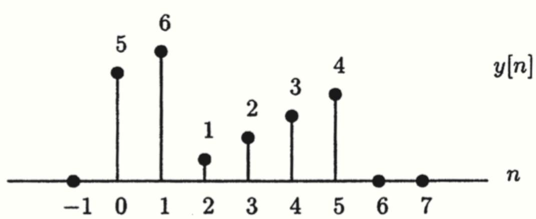
ECE 8440 Spring 2017
Chapter 8 Sample Problems

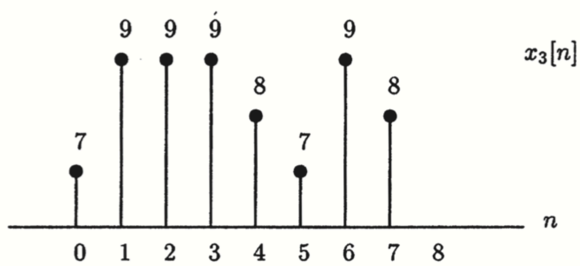
8.11, 8.14, 8.17, 8.28

8.11. We wish to perform the circular convolution between two 6-pt sequences. Since $x_2[n]$ is just a shifted impulse, the circular-convolution coincides with a circular shift of $x_1[n]$ by two points.

$$\begin{aligned} y[n] &= x_1[n] \circledsix x_2[n] \\ &= x_1[n] \circledsix \delta[n-2] \\ &= x_1[((n-2))_6] \end{aligned}$$



8.14. $x_3[n]$ is the linear convolution of $x_1[n]$ and $x_2[n]$ time-aliased to $N = 8$. Carrying out the 8-point circular convolution, we get:



We thus conclude $x_3[2] = 9$.

8.17. Looking at the sequences, we see that $x_1[n] * x_2[n]$ is non-zero for $1 \leq n \leq 8$. The smallest N such that $x_1[n] \circledN x_2[n] = x_1[n] * x_2[n]$ is therefore $N = 9$.

8.28. **(a)** Using the analysis equation

$$X[k]=\sum_{n=0}^{N-1}x[n]W_N^{kn}$$
$$=\sum_{n=0}^{5}x[n]W_6^{kn}$$
$$=6W_6^0+5W_6^k+4W_6^{2k}+3W_6^{3k}+2W_6^{4k}+W_6^{5k}.$$

(b)

$$W[k]=W_6^{-2k}X[k]$$
$$=6W_6^{-2k}+5W_6^{-k}+4+3W_6^k+2W_6^{2k}+W_6^{3k}.$$

Using the fact that $W_6^k=e^{-j\frac{2\pi k}{6}}$,

$$W_6^{-2k}=e^{j\frac{4\pi k}{6}}=e^{j\frac{4\pi k}{6}}\times e^{-j2\pi k}\ \text{(since } e^{-j2\pi k}=1)$$
$$=e^{-j\frac{8\pi k}{6}}=W_6^{4k},$$

and similarly

$$W_6^{-k}=W_6^{5k}.$$

Then

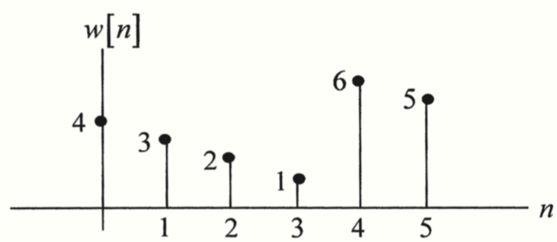
$$W[k]=4+3W_6^k+2W_6^{2k}+W_6^{3k}+6W_6^{4k}+5W_6^{5k}.$$

Using the synthesis equation,

$$w[n]=\frac{1}{6}\sum_{k=0}^{5}W[k]W_6^{-kn}.$$

We could go ahead and solve the problem in this "brute force" method, but notice that each $\delta[n-k]\xrightarrow{DFT}W_N^k$. Then,

$$w[n]=4\delta[n]+3\delta[n-1]+2\delta[n-2]+\delta[n-3]+6\delta[n-4]+5\delta[n-5].$$



Notice that multiplying by W_6^{-2k} in frequency has the effect of a shift of 2 in time, but modulo 6.

(c) One way to do this is to compute the linear convolution and then add copies of it shifted by N (6 in this case). Another method is to use the DFT, find the product $H[k]X[k]$, and then take an inverse DFT. We know

$$X[k] = 6W_6^0 + 5W_6^k + 4W_6^{2k} + 3W_6^{3k} + 2W_6^{4k} + W_6^{5k}$$

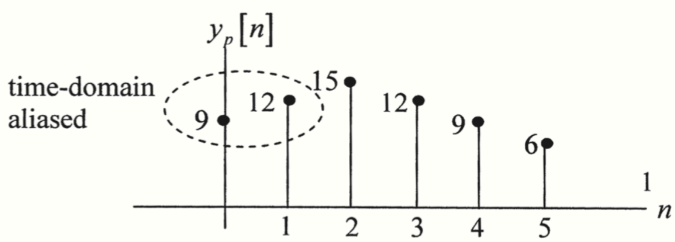
$$H[k] = 1 + W_6^k + W_6^{2k}$$

Then

$$\begin{aligned} Y_p[k] = 6 &+ 5W_6^k + 4W_6^{2k} + 3W_6^{3k} + 2W_6^{4k} + W_6^{5k} \\ &+ 6W_6^k + 5W_6^{2k} + 4W_6^{3k} + 3W_6^{4k} + 2W_6^{5k} + W_6^{6k} \\ &+ 6W_6^{2k} + 5W_6^{3k} + 4W_6^{4k} + 3W_6^{5k} + 2W_6^{6k} + W_6^{7k} \\ &= 9 + 12W_6^k + 15W_6^{2k} + 12W_6^{3k} + 9W_6^{4k} + 6W_6^{5k}, \end{aligned}$$

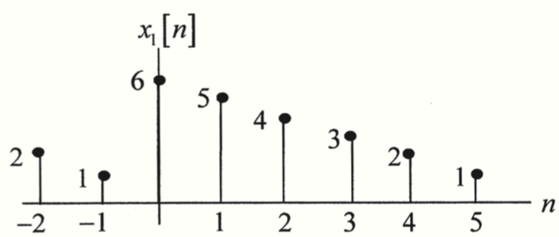
where we have used $W_6^{6k} = 1$ and $W_6^{7k} = W_6^k$. Now we have

$$y_p[n] = 9\delta[n] + 12\delta[n-1] + 15\delta[n-2] + 12\delta[n-3] + 9\delta[n-4] + 6\delta[n-5].$$



(d) To ensure that no time-domain aliasing occurs in the output, N should be large enough to accommodate the length of the linear convolution. That is, $N \geq 6 + 3 - 1 = 8$

(e)



This new input when convolved with $h[n]$ will give the circular convolution found in (c) We merely extend $x[n]$ as a periodic signal with period 6 samples.

(f) In general $x_1[n]$ is constructed by extending $x[n]$ periodically for $n = -1, \ldots, -M$.