Warmup 2: Regular Languages and CFGs

1. (a) State Kleene’s Theorem.

There is FA for language $L \iff$ there is RE for language $L$.

(b) List the three Kleene operators.

union, concatenation, star.

2. Apply the subset construction to produce a DFA that accepts the same language as the following NFA.

3. For each language, give 3 strings that are pairwise distinguishable with respect to that language:

(a) The set of all binary strings whose first and last bit are the same

00, 01, 11

(b) The set of all binary strings that contain 101 as substring

$\epsilon, 1, 101, 101$

(c) The set of all binary strings of odd length.

only two possible:

one of odd length & one of even length
4. For the alphabet \{a, b\}, give a CFG for:

(a) the set of all strings that start and end with abba

$$S \rightarrow \text{abba}T \text{abba} | \text{abba} | \text{abbabba}$$
$$T \rightarrow aT | bT | \epsilon$$

(b) the set of all even-length palindromes that contain abba as a substring.

[abba can be middle or elsewhere]

$$P \rightarrow aPa | bPb | \text{abba}Q \text{abba} | \text{abba}$$
$$Q \rightarrow aQa | bQb | \epsilon$$

5. Consider the following CFG with start variable \( S \):

$$S \rightarrow 0T0 | 1T1 | 0T1 | 1T0 | \epsilon$$
$$T \rightarrow 0S | 1S | \epsilon$$

(a) Give a derivation tree for the string 01010

(b) Describe in English the language of this grammar.

all binary strings except those of length 1