Warmup 3: Context-Free Languages

1. State whether each of the following is true or false. No justification required.
   (a) There exists a language accepted by a DFA but by no PDA. \textit{FALSE}
   (b) There exists a language accepted by a nondeterministic FA but by no deterministic PDA. \textit{FALSE}
   (c) The context-free languages are closed under the three Kleene operations. \textit{TRUE}

2. Give a regular grammar for the language generated by the RE \((x + y)^* (x y y + y x)\)
   
   \[
   S \rightarrow xS \mid yS \mid xA \mid yD \\
   A \rightarrow yB \\
   B \rightarrow yC \mid y \\
   D \rightarrow xE \mid x
   \]

3. Show that the context-free languages are closed under reversal. That is, show algorithmically that if language \(L\) is context-free, then so is \(L^R\), where \(L^R\) consists of the reverses of all strings in \(L\).

   Take the CFG and write each production RHS reversed

   e.g. \(S \rightarrow \emptyset p q \top\) becomes \(S \rightarrow \top q p \emptyset\)
4. Let \( X \) be the set of all binary strings that are odd-length palindromes or all of whose symbols are the same. (For example, 01110 and 111 are in \( X \).) Draw a PDA for \( X \).

5. Consider following PDA.

(a) Give two strings of length 4 accepted by the PDA.

(b) Give two strings of length 4 NOT accepted by the PDA.

(c) Describe in succinct-ish English the language of this PDA. Be precise.

\[
\text{all even-length binary strings of the form } 0^*1^*0^* \text{ where each block of 0's is at most half the string}
\]