Facts about Regular Languages

1. Kleene's theorem says that for a language L the following are equivalent: there is DFA for L, there is NFA for L, and there is RE for L.

2. If A and B are regular languages, then so is:
   (i) the union $A \cup B$, the concatenation AB, the star A^*
   (ii) the intersection $A \cap B$, the complement of A

3. Examples of languages that are not regular include $\mathtt{0}^n\mathtt{1}^n$ and palindromes.

4. There are algorithms for the following decision problems about regular language:
   (i) For input FA/RE M and string w, does M accept/generate w?
   (ii) For input FA/RE M and N, are the languages of M and N the same?
   (iii) For input FA/RE M, is its language empty? is its language everything?

Facts about Context-Free Languages

1. For a language L the following are equivalent: there is PDA for L and there is CFG for L.

2. A PDA by default is nondeterministic. An example of a context-free language that does not have a deterministic PDA is palindromes.

3. If A and B are context-free languages, then so is:
   (i) the union $A \cup B$, the concatenation AB, the star A^*

4. Examples of languages that are not context-free include $\mathtt{0}^n\mathtt{1}^n\mathtt{2}^n$ and palindromes-with-equal-`0`s-and-`1`s. (However, the complement of these two particular languages are context-free.)

5. There are algorithms for the following decision problems about context-free languages:
   (i) For input PDA/CFG M and string w, does M accept/generate w?
   (ii) For input PDA/CFG M, is its language empty?

6. There following decision problems about context-free languages are undecidable:
   (i) For input PDA/CFG M and N, are the languages of M and N the same?
   (ii) For input PDA/CFG M, is its language everything?

Facts about TMs

1. Between context-free and TM on the Chomsky hierarchy are the context-sensitive languages. These have a context-senstive grammar or equivalently an LBA.

2. A TM is defined to be deterministic (but allowing nondeterminism does not change the power). Also the power of a TM is not affected by adding multiple tapes or by adding RAM.

3. Church's thesis says that there is a TM for a problem if and only if there is an effective procedure for it (which we call an algorithm): must always halt and must always be correct.

4. A recursive language is one accepted by a TM that halts on all input. An r.e. language is one accepted by a TM.

5. (i) If A and B are recursive languages, then so is: the union $A \cup B$, the concatenation AB, the star A^*, the intersection $A \cap B$, and the complement of A.
   (ii) If A and B are r.e. languages, then so is: the union $A \cup B$, the concatenation AB, the star A^*, and the intersection $A \cap B$, but NOT necessarily the complement of A.

6. A language is recursive if and only if both it and its complement are r.e.

7. One uses parallelism when one needs to run multiple strings through a TM that is not guaranteed to halt.

8. A decision problem is decidable if the associated language is recursive.

9. Angle brackets convert a decision problem to a language. For example, $\langle M \rangle$ means a string that can be parsed as machine M.

Facts about Limits of Computation

1. S_{tm} is the set of all $\langle M \rangle$ such that M is a TM that does NOT accept $\langle M \rangle$. Language S_{tm} is not r.e.

2. A_{tm} is the set of $\langle M, w \rangle$ where TM M accepts string w. Language A_{tm} is r.e. but not recursive.

3. Diagonalization in a table produces something that is not in the table.

4. The set of all FAs, REs, CFGs, PDAs, or TMs is countable. The set of languages is uncountable.

5. If language A reduces to language B, and B is recursive, then so is A. To show that B is not recursive we reduce a non-recursive language to it. Similarly with r.e.

6. Most (but not all) decision problems about TM languages are undecidable. These include
   (i) For input TM M and string w, does M accept w ?
   (ii) For input TM M and string w, does M halt on w ? (The Halting Problem)
   (iii) For input TM M, is its language empty ?
   (iv) For input TMs M and N, do they have the same language ?