Assignment 4

(Please work in groups of two or three and submit one answer sheet for the group.)

1. Let $\Sigma = \{0, 1, 2\}$. For a string $w$ with alphabet $\Sigma$, the aab-fication of $w$ is defined by replacing every 0 by the string aab, every 1 by the string a and every 2 by the string ab. For a language $L$ whose alphabet is $\Sigma$, the aab-fication of $L$ is obtained by taking the aab-fication of every string in $L$. For example, if 012 is in $L$, then aabaab is in the aab-fication of $L$. Prove that the regular languages are closed under aab-fication.

2. For each of the following languages, give a biggest possible set of pairwise distinguishable strings with respect to the language.
   (a) The set of all binary strings ending in 01.
   (b) The set of all binary strings with more 0s than 1s.

3. Determine, with justification, whether each of the following is true or false:
   (a) There exist languages $A_1$ and $A_2$ that are regular but their union $A_1 \cup A_2$ is not regular.
   (b) There exist languages $B_1$ and $B_2$ that are not regular but their union $B_1 \cup B_2$ is regular.
   (c) There exists an infinite language $C$ that is not regular but every finite subset of $C$ is regular.

4. Construct a CFG for the language of binary strings of the form $0^n1^{2n}$.

5. For each of the following CFGs: (1) determine which of the strings $\varepsilon$, abba, aaaaa and b is generated, and (2) give an English description of the language.

   (a) $S \rightarrow aS | bS | a | b | \varepsilon$
   (b) $S \rightarrow XaaaX$
      $X \rightarrow aX | bX | \varepsilon$
   (c) $S \rightarrow aaS | aaaS | a$
   (d) $S \rightarrow aX | bS | a | b$
      $X \rightarrow aX | a$

6. Let $E$ be the language generated by the regular expression $(00)^*(11)^*$. Give a CFG for $E$.

Due: Tuesday September 27