Warmup 2: Regular Languages and CFGs

[ about $\frac{3}{4}$ length of actual test ]

1. (a) State Kleene’s Theorem.

   there is FA for language L $\iff$ there is RE for language L

   (b) List the three Kleene operators.

   union, concatenation, star

2. Given a string $x$, an expansion of $x$ is any string obtained by repeating some of the letters some number of times. For example, each of CCAATT, CAT and CCCCCCAAT are expansions of CAT. Given a language $L$, the expansion of $L$ is all possible expansions of strings in $L$. Show that the regular languages are closed under expansion.

   e.g. transform RE by replacing each char $x$ by the expression $xx^*$

   e.g. $(0+11)0 \Rightarrow (00^*+11^*11^*)00^*$

3. For each language, give 3 strings that are pairwise distinguishable with respect to that language:

   (a) The set of all binary strings whose first and last bit are the same

      e.g. 11, 01, 00 [first & last bit matter]

   (b) The set of all binary strings that contain 101 as substring

      e.g. $\epsilon$, 1, 10 [progress on containing 101]

   (c) The set of all binary strings of odd length.

      can only find 2, one of even length & one of odd length.
4. For the alphabet \{a, b\}, give a CFG for:

(a) the set of all strings that start and end with \textit{abba}

\[
S \to \textit{abba} T \textit{abba} | \textit{abba} | \textit{abbabba} \\
T \to aT | bT | \epsilon
\]

(b) the set of all even-length palindromes that contain \textit{abba} as a substring.


\[
\begin{align*}
P & \to aPa | bPb | \textit{abba} Q \textit{abba} | \textit{abba} \\
Q & \to aQa | bQb | \epsilon
\end{align*}
\]

5. Consider the following CFG with start variable \textit{S}:

\[
S \to 0T0 | 1T1 | 0T1 | 1T0 | \epsilon \\
T \to 0S | 1S | \epsilon
\]

(a) Give a derivation tree for the string \textit{01010}

(b) Describe in English the language of this grammar.

\textit{all binary strings except those of length 1}