Warmup 3: Context-Free Languages

[ about \( \frac{3}{4} \) length of actual test ]

1. Give a regular grammar for the language generated by the RE \((x+y)^*(xxy+yx)\):

\[
S \rightarrow xS | yS | xA | yD \\
A \rightarrow yB \\
B \rightarrow yC | y \\
D \rightarrow xE | x
\]

2. Consider following PDA.

(a) Give two strings of length 4 accepted by the PDA.

(b) Give two strings of length 4 NOT accepted by the PDA.

(c) Describe in succinct-ish English the language of this PDA. Be precise.

all even-length binary strings of the form \(0^*1^*0^*\) where each block of 0's is at most half the string
3. Show that the context-free languages are closed under reversal. That is, show algorithmically that if language $L$ is context-free, then so is $L^R$, where $L^R$ consists of the reverses of all strings in $L$.

Take the CFG and write each production reversed.

\[ S \rightarrow \phi PQ1 \text{ becomes } S \rightarrow 1QP\phi \]

4. Let $EP$ be the language of all binary palindromes that have an equal number of 0’s and 1’s. Suppose one has to prove, using the Pumping Lemma, that $EP$ is not regular. Assume $k$ is the constant of the Pumping Lemma. For each of the following strings, state whether it is suitable for the string $z$ that leads to a contradiction in the Pumping Lemma. Justify your answer.

(a) $0^k 1^k$

Not suitable. Not in language.

(b) $0^k 1^{2k} 0^k$

Suitable. If we write $z = uvvw$ with $|uvw| \leq k$ then $uv^jw$ is a string of 0’s, and so $u^j v^j w$ is neither palindrome nor equal 0’s & 1’s.

(c) $(10)^{k/2}(01)^{k/2}$

Not suitable. If we write $z = uuvw$ with $|uv| = 1001$, the last 10 and first 01, then we can pump '10 and remain in the language and so NOT get contradiction.

(d) $0^{2022} 1^{2022}$

Not suitable. Not in language & not guaranteed long enough.