A Printer Turing Machine

A ***printer-TM*** is TM with an added printer-tape. The printer-TM writes strings on the printer-tape (separated by Δ); once written, a string is not altered.

Theorem. *A language is r.e. if and only if some printer-TM outputs precisely those strings.*

The proof is in two constructions. . .

From Printer-TM to Standard TM

Armed with printer-TM M for language L, we build standard TM N.

On input x, TM N runs M and monitors M; if N ever finds x on the printer-tape, then N accepts. So N accepts strings in L, and does not halt otherwise.

From Standard TM to Printer-TM

Armed with standard TM N for L, we build printer-TM M.

The idea is to run N on every possible string in parallel—an infinite number of tasks!—and print out those it accepts.

Infinite Parallelism

The printer-TM works M in rounds.

In round i, M starting from scratch, generates the first i strings lexicographically (in dictionary order), runs N on each for i steps, and outputs any string that is accepted.

Eventually, every string in $L(N)$ will be generated and N run for long enough, and will appear in the output.