Closure

Theorem If A, B regular languages, then so are

1. $A \cup B$ (union)
2. AB (concatenation)
3. A^*
4. $A \cap B$ (intersection)
5. $\overline{A}$ (complement)

In other words, the class of regular languages are **closed** under union, concatenation, star, intersection and complement.

PROOF:

1–3 Build FA for A and B. Then convert to FA for $A \cup B$ OR AB OR A^* as in algorithm RE to FA.

$\boxed{\text{OR}}$ Build RE for A and B. Then combine:

$R_A + R_B$ OR $R_A R_B$

OR $(R_A)^*$

4 product construction

5 For DFA, interchange accept & reject

de Morgan's law

$$A \cap B = \overline{\bar{A} \cup \bar{B}}$$

So if closed under union and complementation, then must be closed under intersection.

<u>Theorem</u> Regular languages closed under reversal.

In other words, if A is regular, then language of all reversals of ~~words~~ strings in A is also regular.

Idea: Reverse RE OR

Reverse arrows on FA.