Pumping Lemma for Regular Languages

If language L is regular, then there is constant k such that every string in L of length at least k is <u>pumpable</u>

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We show a language is not regular by giving one long string in L and showing that that string is not pumpable

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A string is <u>pumpable</u> if it can be split into uvw such that uw, $uvvw$, $uvvvw$... are all in the language.

Require that $v \neq \varepsilon$ and $|uv| \leq k$ (length)

Example Let $L = 0^n 1^n$.

Then $z = 0^k 1^k$ is not pumpable.

Because $z = uvw$ with $|uv| \leq k$ forces v to be a string of 0's, and then uw is not in L.

Example Let $L =$ equal 0's and 1's

Then $z = (01)^k$ is pumpable.

Because we can choose $v = 01$ and uv^iw is in L for $i \geq 0$.

Example Let $L =$ odd 0's and even 1's

Then $z = 0^{2k+1} 1^{2k}$ is pumpable.

Because we can choose $v = 00$ and uv^iw is in L for $i = 0, 1, 2, \ldots$

Example Let P = binary palindromes.
Prove that P is not regular.

Proof

Suppose P is regular.
Let k be constant of pumping lemma.
Let $z = 0^k 1^k 0^k$
Then $|z| \geq k$ and z in P.

Consider split $z = uvw$ $|uv| \leq k$
then v is part of first block of 0's. So uw is not a palindrome. That is, z is not pumpable, a contradiction

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All uses of Pumping Lemma look like above.