Pumping Lemma for Context-Free

The pumping lemma for context-free languages is the same as for regular languages except the definition of split changes.

Now a string z is *pumpable* with respect to language L if can write $z = uvwxy$ such that uv^iwx^iy is in L for all $i \geq 0$.

Require that $vx \neq \varepsilon$ and $|vwx| \leq k$.

Example Let $L = 0^n 1^n 2^n$

Then $z = 0^k 1^k 2^k$ is not pumpable.

Because $z = uvwxy$ with $|vwx| \leq k$ mearse vx cannot contain all 3 characters.

So uwy cannot have equal numbers of the 3 characters and is not in L.

Example Let $M = \{x\#y : x$ & y binary with x initial segment of $y\}$

Show that M not context-free

Proof:

Suppose M context-free.

Let k be constant of pumping lemma.

Let $z = 0^k 1^k \# 0^k 1^k$

Note $|z| \geq k$ and z in M.

Consider split $z = uvwxy$
with $|vwx| \le k$.

If v contains 0 from first block,
then x does not contain 0 from second block
so $uv^k wx^k y$ is not in L

If v contains 1 from first block
then x does not contain 1 from
second block
so $uv^k wx^k y$ is not in L.

If v does not contain anything
to the left of the hash, then
uwy is not in L.

That is, z is not pumpable.

Practice: Show $\{a^i b^j c^i d^j\}$
is not context-free.