

DIAGONALISATION

collection of strings arranged in

Consider a square grid of symbols.
For example

S H A R K
C H E R I
N I F T Y
P A N D A
Q Q Q Q Q

The idea: create a string by
incrementing 1st letter of 1st word
incrementing 2nd letter of 2nd word
...
incrementing last letter of last word

~~S H A R K
C H E R I
N I F T Y
P A N D A
Q Q Q Q Q~~

TIGER

Yield string NOT in the original collection
Because it's different to each one.

DIAGONALISATION WITH TMS

Consider a list of all TMs. Each can be represented by string. So for example we might list in increasing length.

Whatever. We get $M_1, M_2, M_3, \dots$

Create a table that says:

Is string $\langle M_j \rangle$ in language of M_i ?

	$\langle M_1 \rangle$	$\langle M_2 \rangle$	$\langle M_3 \rangle$	$\langle M_4 \rangle$	$\dots$
M_1	✓	✓	✓	✓	
M_2	x	x	x	x	
M_3	✓	x	x	✓	
M_4	✓	x	✓	✓	

Handwritten annotations on the table:

- A red diagonal line runs from the top-left to the bottom-right.
- Red "YES" is written above the cell $(M_2, \langle M_2 \rangle)$.
- Red "YES" is written above the cell $(M_3, \langle M_3 \rangle)$.
- Red "NO" is written to the right of the cell $(M_4, \langle M_4 \rangle)$.
- Red "NO" is written above the cell $(M_1, \langle M_1 \rangle)$.

Table is infinite. Now do diagonalization to create language. Take $\langle M_1 \rangle$ if first row first column is x. Take $\langle M_2 \rangle$ if second row second column is x. And so on. That is, take diagonal and invert it.

let X be the new language.

It is not the language of M_1 , because they differ on $\langle M_1 \rangle$.

It is not the language of M_2 , because they differ on $\langle M_2 \rangle$.

And so on.

Drum Roll Please

X is not a language accepted by any TM

But we've seen X before.

It's, wait for it,
wait for it, S_{TM} .

That is, we've shown that S_{TM} is not r.e.