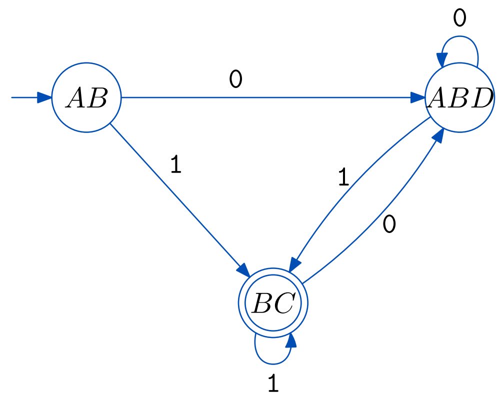
Some Answers on: Regular Languages

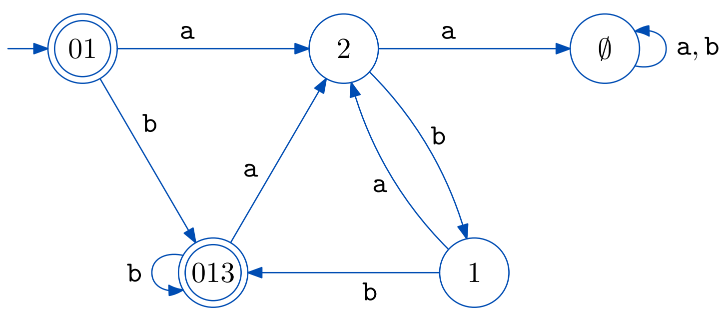
B1: (a)



(b) Several answers including: $(\mathtt{b}^*\mathtt{a} + \mathtt{bab}^*\mathtt{a})^*\mathtt{b}$

B2: Use the product construction with the two-state DFA accepting all string of even length. If M is the FA for L, then the FA for L^E is as follows. For every state q in M, create two states labeled q^e and q^o. The start state is q_0^e where q_0 was the start state of M; the accept state are all states q^e where q is an accept state of M. And for each transition $q_i \to q_j$ of M add two transitions $q_i^e \to q_j^o$ and $q_i^o \to q_j^e$.

B3:



B4: (a) Add a new start state with ε-transitions to all other states.

(b) Add a separate copy of the automaton, say L'. Then for every non-ε-transition in L, say $x \to y$ on c, add an ε-transition from x to the copy of y in L'. Finally, make all states in L reject.

(c) For every state, say x, create two copies, say x_1 and x_2. For every original transition $x \to y$ on c, add a transition from x_1 to y_2 on c and a transition from x_2 to y_1 on ε.

B5: (a) Let L_1 be any nonregular language and let L_2 be its complement.
(b) $\{\mathtt{0}^{100}\}$

B6: (a) A DFA accepts every string if and only if every state is an accept state.
(b) Convert the RE to a DFA (via an NFA) and then use the above test.

B7: (a) $\{\varepsilon, \texttt{1}, \texttt{11}, \texttt{110}\}$
(b) $\{\varepsilon, \texttt{0}, \texttt{00}, \texttt{000}\}$
(c) $\{\varepsilon, \texttt{0}, \texttt{01}, \texttt{011}, \texttt{1}\}$

B8: Suppose the language, call it L, were regular. Let k be the number of states of a DFA for L. Consider the string $z = \texttt{0}^k\texttt{1}^k\texttt{0}^k$—this is in L. Split $z = uvw$ according to the Pumping Lemma. Then, because $|uv| \leq k$, it follows that v is always a string of 0's. Thus, $uv^2w \notin L$, a contradiction of the Pumping Lemma.

B9: Let C be an infinite subset of B and suppose that C is regular. Then C is accepted by some DFA with k states. Since C is infinite, there exists some $\ell \geq k$ such that $z = \texttt{0}^\ell\texttt{1}^\ell$ is in C. By the Pumping Lemma one can write $z = uvw$ such that v is nonempty and uv^2w is in C. But uv^2w either has the wrong format or does not have equal numbers of 0s and 1s, a contradiction. Therefore C is not regular.

B10: $\{\varepsilon, \texttt{0}, \texttt{1}, \texttt{00}\}$