# Some Answers on: Context-Free Languages

D1: (a) palindromes without middle marker
(b) $0^n1^n$
(c) $0^n1^n2^n$

D2: (b)
$A \to 0C \mid 1B$
$B \to 0D \mid 1A \mid 0$
$C \to 0A \mid 1D \mid 1$
$D \to 0B \mid 1C$

D3: (a) No
(b) No
(c) Yes: the language contains just one string: it is $\{a\}$.

D4: (a) Any context-free language e.g. $\Sigma^*$
(b) Does Not Exist
(c) For example, $C = \{0, 1, 2\} \cup \{ 0^n1^n2^n : n \geq 0 \}$: here $C^* = \Sigma^*$
(d) $0^n1^n2^n$
(e) $0^n1^n2^m$ and $0^n1^m2^m$
(f) Take any two non context-free languages whose alphabets are disjoint.

D5: (a)

$S \to 0S \mid 1S \mid 0T$
$T \to 0$

(b)

$S \to 0S \mid 0T \mid 0$
$T \to 0S$

D6: Take the CFG for $L$. Then in every production, replace a by 0, b by 1, and c by 01.

D7: Assume $B \cap E$ is context-free. Let $k$ be the constant of the Pumping Lemma. Let $z = 0^k1^{2k}0^k$. Note $z \in L$.
Consider split $z = uvwxy$. Since $|vwx| \leq k$, if $v$ or $x$ contains 0's, then these 0's are from only one block of 0's, and so $z^{(2)} = uv^2wx^2y$ is not a palindrome. On the other hand, if $v$ and $x$ contain only 1's, then $z^{(2)} = uv^2wx^2y$ has more 1's than 0's.
A contradiction.

D8: 1) We should assume the language IS context-free.
2) We did not guarantee $z$ is long enough for the Pumping Lemma to apply.
3) We did not consider all possibilities for $v$.

D9: Let $L$ stand for the language. Suppose that $L$ is context-free. Let $k$ be the constant of the pumping lemma. Set $z = \mathtt{a}^k\mathtt{b}^k\mathtt{a}^k\mathtt{b}^k\mathtt{a}^k\mathtt{b}^k$. Say one writes $z = uvwxy$. The pumping lemma claims that $z^{(i)} \in L$ for all $i$; but this does not hold for $z^{(0)}$. For, since $|vwx| \leq k$, at least one block of a's is undisturbed as one block of b's. So the result when split in three does not have three identical pieces. This is a contradiction. Therefore, $L$ is not context-free.