A Proof that A_{tm} is not recursive

One Small Step for Man

It's a proof by contradiction. Suppose there were machine H that always halted and accepted A_{tm}. That is, on *every* input $\langle M, w \rangle$ the machine H would halt and would tell one whether or not M accepted w. Then...

Building a New TM

Build a new TM D that does the following:

D: On input w

1. Determine the TM S that w encodes.
2. Run H on the pair $\langle S, w \rangle$.

   Note that we are supposing H always halts.
3. If H accepts then reject; else accept.

The Million Bitcoin Question

But wait. What happens if the input is the description of D, say $w' = \langle D \rangle$ (not that D notices)?

A Contradiction

Well, D writes $\langle D, w' \rangle$ on the tape, and feeds to H. If H says accept, then D rejects, and vice versa.

That is, if H claims that D accepts $w' = \langle D \rangle$, then D rejects w'. If H says D rejects $w' = \langle D \rangle$, then D accepts w'. Huh?

This is a contradiction. Everything we did was fine except possibly that H exists. Conclusion: H does not exist.