Supplemental Questions on: Context-Free Languages

D1: Give an example of:

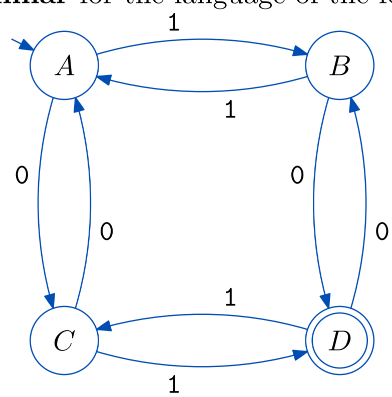
(a) a language accepted by a PDA but not by any deterministic PDA.

(b) a language accepted by a deterministic PDA but not by any NFA.

(c) a non-context-free language whose complement is context-free.

D2: (a) Define a regular grammar.

(b) Give a **regular grammar** for the language of the following FA.



D3: For the following grammar:

$$S \to AS \mid SB \mid \texttt{a}$$
$$A \to BA$$
$$B \to \varepsilon$$

(a) Does this grammar generate the empty string?

(b) Is this grammar in Chomsky Normal Form?

(c) Is the language of this grammar finite?

D4: Give one example of each of the following, or state that it does not exist.

(a) A context-free language A such that A^* is context-free.

(b) A context-free language B such that B^* is **not** context-free.

(c) A **non**-context-free language C such that C^* is context-free.

(d) A **non**-context-free language D such that D^* is **not** context-free.

(e) Languages E_1 and E_2 such that E_1 and E_2 are context-free but $E_1 \cap E_2$ is not.

(f) Languages F_1 and F_2 such that F_1 and F_2 are not context-free but $F_1 \cap F_2$ is.

D5: (a) Give an **unambigous regular** grammar for the set of all binary strings ending in `00`.

(b) Give an example of a regular grammar that is ambiguous.

D6: For a string w from the alphabet $\{\texttt{a}, \texttt{b}, \texttt{c}\}$, the **splonk** of w is obtained by replacing every `a` by `0`, every `b` by `1`, and every `c` by `01`. For example, if w is `abbc`, then the splonk of w is `01101`. For a language L with alphabet $\{\texttt{a}, \texttt{b}, \texttt{c}\}$, let L^s be the set of splonks of all strings in L.

Show, by means of an algorithm, that if L is context-free then so is L^s.

D7: Let B denote the set of all binary palindromes and E denote the set of all strings with equal numbers of 0s and 1s. Use the Pumping Lemma to show that the intersection $B \cap E$ is not context-free.

D8: Explain TWO things wrong with the following "proof" that the language L of $\mathtt{0}^n\mathtt{1}^n$ is not context-free.

> *Assume L is not context-free.*
> *Consider the string $z = \mathtt{0}^{999}\mathtt{1}^{999}$.*
> *Then z is in L.*
> *Split z as $z = uvwxy$.*
> *Assume $v = \mathtt{01}$ and $x = \varepsilon$.*
> *Then uv^2wx^2y is not in L.*
> *This is a contradiction of the Pumping Lemma, and so L is not context-free.*

D9: Show that the following language is not context-free: $\{\, www : w \in \{\mathtt{a}, \mathtt{b}\}^* \,\}$