Self-Denial

Define the self-denying machines by

$$S_{tm} = \{ \langle M \rangle : M \text{ is TM not accepting } \langle M \rangle \}$$

As before $\langle M \rangle$ means M encoded/represented as a binary string.

Note that S_{tm} is a valid language. If we were omniscient, we could see whether w is in S_{tm} by parsing w as a TM, and then checking whether that TM accepted w.

The Language S_{tm} is Not R.e.

Self-denial. *S_{tm} is not r.e.*

Proof by Contradiction. Suppose some TM accepts this language: call it M'.

Does M' accept string $\langle M' \rangle$? Well, $\langle M' \rangle$ is in S_{tm} if and only if M' does not accept $\langle M' \rangle$. That is, M' accepts $\langle M' \rangle$ if and only if M' does not accept $\langle M' \rangle$.

A contradiction. Since the logic is correct, the problem is the supposition: M' does not exist.