

In-class Practice 11: Context-Free Grammars

For the alphabet $\{0, 1\}$, give a CFG for each language:

1. All strings where the 0s and 1s alternate.

$$\begin{aligned} S &\rightarrow 0A \mid 1B \mid \varepsilon \\ A &\rightarrow 1B \mid \varepsilon \\ B &\rightarrow 0A \mid \varepsilon \end{aligned}$$

2. All palindromes where the number of 0's is a multiple of 4.

The idea is to use two variables: one for palindromes where the number of 0's is a multiple of 4 and one where the number of 0's is even but not a multiple of 4. So CFG is

$$\begin{aligned} P &\rightarrow 0Q0 \mid 1P1 \mid \varepsilon \mid 1 \\ Q &\rightarrow 0P0 \mid 1Q1 \end{aligned}$$

3. All strings where the difference between the number of 0s and the number of 1s is odd.

The property is equivalent to the string having odd length.