CpSc 3500 — Goddard

In-class Practice 21: Closure Again

Let A be an alphabet, and let f be a function that maps each symbol in A to some nonempty string. Given a string w in A^*, we define the string w^f as replacing every symbol in w by its corresponding f value. And we define for language L the language L^f as the set of all w^f for w in L.

(a) Show that if L is r.e. then so is L^f.

> Here is procedure for L^f.
> Consider the input string x.
> Generate all possible strings w such that $w^f = x$.
> (Since f does not replace symbol by empty string, the length of w is at most the length of x; so one can just generate all strings of length at most $|x|$ from alphabet A and see which of them work.)
> For all the w such that $w^f = x$:
> run machine for L on the string w, doing all of this in parallel.
> If any of the machines accept their w, stop and accept.
> If all the machines terminate and reject their w, reject.

(b) Show (by means of an example) that L^f can be regular even if L is not.

> Consider your favorite non-regular binary language L that contains a string of all possible lengths.
> E.g. binary strings with unequal `0`'s and `1`'s.
> Then let f map both `0` and `1` to the symbol `#`.
> Then L^f is all strings of `#`, which is clearly regular.