CpSc 3500 — Goddard

In-class Practice 22: Decidable Problems

Consider an Enhanced-FA as an FA that can move left or right on the input (equivalently a TM that is not allowed to write on the tape nor to move off the input).

Show that the acceptance problem for Enhanced-FA is decidable.

Say the input w has length n and the Enhanced-FA has k states. We claim that if w is accepted, then the shortest accepting computation branch has length at most kn.

For, assume w is accepted and look at shortest accepting computation branch. Suppose the branch has length more than nk steps. Then somewhere we must repeat the same state and head position; say at time i and time j. Then the computation between i and j can be removed and we still have an accepting computation, contradicting the claim that this was the shortest branch.

Thus run Enhanced-FA for $nk + 1$ steps through all possibilities. If no branch reaches acceptance within that time, then input w is not in the language. If some branch reaches acceptance within that time, then input is in the language.