Math 3110 — Goddard — Fall22
Assignment 3

(Please work in groups of two or three and submit one answer sheet for the group.)

1. Wanja started with the (un-augmented) matrix \( C \) below, and using elementary row operations reached the matrix \( D \) shown. Answer the following with brief justification.

\[
C = \begin{bmatrix}
1 & 2 & 3 & -4 \\
2 & 5 & 7 & 3 \\
13 & 30 & 43 & -8 \\
4 & 10 & 14 & 6
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
1 & 0 & 1 & -26 \\
0 & 1 & 1 & 11 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

(a) Does the matrix equation \( Cx = 0 \) have a solution?
(b) Does the matrix equation \( Cx = b \) have a solution for every \( b \) ?
(c) Is the span of the columns of \( C \) all of \( \mathbb{R}^4 \) ?
(d) Is there a \( b \) such that the matrix equation \( Cx = b \) has a unique solution?

2. Show that each of the following sets are linearly dependent by finding a nontrivial linear combination that equals the zero vector.

(a) \((1, 0), (2, 0), (0, 1)\)
(b) \((1, 2, 3), (4, 0, -1), (-5, 6, 11)\).

3. For each triple of vectors, find the value(s) of \( h \) so that that triple of vectors is linearly independent.

(a) \((1, 1, 1), (1, 1, 2), (1, 1, h)\)
(b) \((0, h, h), (h, h, 0), (h, 0, h)\)
(c) \((1, 1, 1), (2, 3, 4), (5, h, h)\)

4. Define the function \( g \) that maps a vector in \( \mathbb{R}^2 \) to a number by the rule \((x, y) \mapsto 3x - y\).

(a) Prove that for all vectors \( u \) and \( v \) it is true that \( g(u + v) = g(u) + g(v) \) and \( g(42u) = 42g(u) \).
(b) Determine all \( u \) such that \( g(u) = 0 \).
(c) Describe your answer to (b) geometrically.

5. Let matrix \( F = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \). Let vector \( x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) and for \( i \geq 0 \) define vector \( x_{i+1} = Fx_i \).

(a) Calculate \( x_1 \) up to \( x_5 \).
(b) What does \( F \) stand for?

Due: Monday September 19
Please submit on Canvas as a single pdf