Math 3110 — Goddard — Fall22

Assignment 6

(Please work in groups of two or three and submit one answer sheet for the group.)

1. Let $G_n$ be the $n \times n$ matrix that is 1’s on and above the SW-NE diagonal (not the standard NW-SE diagonal) and 0’s below it. For example,

\[
G_2 = \begin{bmatrix}
1 & 1 \\
1 & 0
\end{bmatrix} \quad \text{and} \quad G_3 = \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 0 \\
1 & 0 & 0
\end{bmatrix}
\]

(a) What is the determinant of $G_2$?
(b) What is the determinant of $G_3$?
(c) What is the determinant of $G_{101}$?

2. State whether each of the following sets is closed under addition and/or scalar multiplication. Justify your answer.

(a) The span of vectors $(1,1)$, $(-2,3)$, and $(0,5)$.
(b) All vectors in $\mathbb{R}^3$ whose entries are all positive.
(c) All vectors in $\mathbb{R}^4$ whose entries sum to 0.

3. For each of the following subsets of $\mathbb{R}^4$, determine with justification whether it is a subspace.

(a) All vectors whose entries multiply to zero.
(b) The set of all linear combinations of vectors $(1,1,2,2)$ and $(2,2,4,4)$.
(c) All vectors whose components are integers.

4. In $\mathbb{R}^3$, state whether each of the following is a subspace. Justify your answer.

(a) The x-axis.
(b) The span of vectors $(1,1,1)$, $(-2,3,4)$, and $(0,5,6)$.
(c) All vectors whose entries are all positive.
(d) All vectors whose second entry is a 0.

5. Prove that if $U$ and $V$ are subspaces neither of which is contained within the other, then their union is guaranteed to not be a subspace.

Due: Monday October 17
Please submit on Canvas as a single pdf