1. Complete the following characterization of invertible matrices: A square matrix is invertible if and only if
   (a) it is row equivalent to... the identity
   (b) the columns of $A$ are... linearly independent
   (c) the determinant of $A$ is... not zero

2. Consider the following matrices

   $$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 3 & 5 \end{bmatrix}$$

   Determine:
   (a) the determinant of $A$.
   
   $$3$$

   (b) the determinant of $B$.
   
   $$8 \times 1 = 8$$

   (c) the determinant of the matrix obtained from $A$ by multiplying every entry by 100.
   
   $$3 \times 100^3$$

   (d) the determinant of $B^{1000}$.
   
   $$8^{1000}$$
3. Let us say a $2 \times 2$ matrix $B$ is **purple** if $B^2 = B$.

(a) Give two examples of singular purple matrices.

\[
\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}
\]

(b) Show that there are exactly two possible values for the determinant of a purple matrix.

\[
\det(B^2) = \det(B) \times \det(B)
\]

but $B^2 = B$. So need $\det(B)^2 = \det(B)$

So $\det(B) = 0$ or $1$

(c) Find all invertible purple matrices.

\[
B^2 = B \implies BBB^{-1} = BB^{-1} \implies B = I
\]

4. Determine whether each of the following is True/False. Justify your answer.

(a) The real numbers are closed under division.

   **False** (No divide by zero)

(b) The set of all polynomials with an even number of terms (such as $0$ or $1 + t$) forms a vector space.

   **False** : $(1 + t) + (-1 + t) = 2t$

(c) $\mathbb{R}^3$ is a subspace of $\mathbb{R}^4$.

   **False !** ($\mathbb{R}^3$ is not contained in $\mathbb{R}^4$)

(d) The row space of the identity matrix is the same as its column space.

   **True** (both $\mathbb{R}^n$)

(e) The union of two subspaces is always a subspace.

   **False** (e.g. in $\mathbb{R}^2$ two lines are subspaces but their union is not)