1. Show that if \( p \) is a prime number, then \( \binom{p}{i} \) is a multiple of \( p \) for all \( i \) from 1 up to \( p - 1 \).

2. Using the Binomial Theorem (and without using Fermat’s Little Theorem), prove that for any odd prime \( p \), it holds that \( 2^p \mod p = 2 \).

3. Prove that \( \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n} \) for all \( n \geq 1 \).

4. Prove that \( n! > 2^n \) for \( n \geq 4 \).

5. In a rabbit warren, each pair of rabbits aged two months or more produces 2 pairs per month (and never dies).
   (a) Give a recurrence for the number of pairs after \( n \) months.
   (b) If we start with 1 newborn pair, how many rabbits do we have after one year?

6. Prove that the Fibonacci sequence obeys the following identity:
   \[ f(0) + f(1) + \ldots + f(n) = f(n+2) - 1. \]

Due: Monday October 30