1. Wayne has a membership at the FoolTiltPoker website that allows him to play any days that he likes, except that he can never play on three consecutive days. Let \( p(n) \) be the number of possible subsets of days in an \( n \)-day period when Wayne can play.

(a) Explain why \( p(3) = 7 \).
(b) Calculate \( p(4) \).
(c) Give a recurrence for \( p(n) \).

2. Draw a rooted tree with 5 vertices labeled \( A, B, C, D, \) and \( E \) such that all of the following conditions hold:
   (i) \( E \) has exactly two ancestors
   (ii) \( A \) and \( C \) are siblings
   (iii) No vertex has exactly one child
   (iv) \( B \) is neither an ancestor nor a descendant of \( C \)
   (v) \( B \) is not the grandparent of \( D \)

3. (a) Show that there are exactly two trees on 4 vertices if the vertices are indistinguishable.

(b) How many different trees are there with 4 vertices if the 4 vertices are all distinguishable (say the vertices are labeled \( A, B, C, D \))?

(c) How many different rooted trees are there with 4 vertices if the 4 vertices are indistinguishable (and the ordering of children does not matter)?

(d) How many different rooted trees are there with 4 vertices if the 4 vertices are all distinguishable (but the ordering of children does not matter)?

4. An irreducible tree is one that has no vertex of degree 2.

(a) According to OEIS A000014, how many such trees are there with 10 vertices.

(b) Draw all irreducible trees with 7 vertices.

5. (a) I have a graph with 2023 vertices. It is not a tree but there exists some vertex whose removal makes it a tree. What is the maximum possible number of edges in the original graph?

(b) I have a graph with 2023 vertices. It is not a tree but if any vertex is removed what remains is a tree. What is the maximum possible number of edges in the original graph?

Due: Monday November 6