1. Draw all (simple) graphs with 5 vertices and 7 edges.

2. Define the graph $G_m$ as the graph that is a grid of $3m$ vertices arranged in 3 rows and $m$ columns such that each vertex has an edge to the vertices to the left, above, to the right, and below it, if they exist. For example, $G_4$ is illustrated here.

Complete the following, with justifications:

(a) $G_m$ has an Euler tour if and only if
(b) $G_m$ has a Hamilton cycle if and only if

3. A **tournament** is obtained by taking the complete graph $K_n$ and orienting every edge to form a directed graph (where every road is a one-way street).

(a) Show that a tournament always has a directed Hamilton path.
(b) Show that a tournament might not have a directed Hamilton cycle.

4. Calculate the chromatic number of the following graph:

5. Show that both $K_5$ and $K_{3,3}$ can be drawn in the plane with only one pair of edges crossing.

6. A graph is called $k$-degenerate if the graph, and every subgraph of it, has a node of degree at most $k$. For example, trees are 1-degenerate.

(a) Prove that every $k$-degenerate graph has chromatic number at most $k + 1$.
(b) Draw a 2-degenerate graph that is not planar.
(c) Show that all planar graphs are 5-degenerate. (Hint: use Lemma 15.1 and Theorem 16.8.)

Due: Monday November 20