The principle of *mathematical induction*:

**If** statement $p(b)$ is true, and statement $p(n - 1) \Rightarrow p(n)$ is true for all $n > b$, **then** $p(n)$ is true for all integers $n \geq b$. 
Induction Recipe

We first prove the **base case**.

Then we assume $p(n - 1)$ is true—called the **inductive hypothesis** and abbreviated IH—and using this fact, we prove that $p(n)$ is true.
An **arithmetic progression** is where every two consecutive entries differ by the same amount.

A **geometric progression** is a sequence where every two consecutive entries have the same ratio.
The Fibonacci numbers

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ... 

are generated by $f(0) = 1$, $f(1) = 1$, and

$$f(n) = f(n - 1) + f(n - 2).$$

We prove things about Fibonacci numbers using induction.
Fibonacci Counts

Fibonacci numbers count many things, such as:

• number of ways of tiling a $2 \times n$ checkerboard with dominoes.

• number of pairs of rabbits after $n$ months if each pair produces one new pair every month after their first month.

• number of sequences of 0s and 1s of length $n$ without consecutive 1s.
A *recurrence* for a sequence expresses the next term as a function of the previous terms.
A **rooted tree** has **vertices** and **edges** with one vertex designated the **root** and all other vertices having a unique parent.

Rooted trees are normally drawn with the root at the top.

A **tree** is just like a rooted tree, except it does not have a special vertex.
Properties of Trees

(1) A tree is connected
(2) A tree contains no cycle
(3) Between any two vertices in a tree there is a unique path.
(4) If a tree has $n$ vertices, then it has $n - 1$ edges.
A (simple) **graph** is a collection of vertices and edges such that each edge joins two vertices.

We do NOT allow **multiple edges** nor **self-loops**.
A **walk** is a sequence of vertices such that consecutive vertices are joined by an edge. The **length** of a walk is the number of edges.

A **path** is a walk without repeated vertices. A **cycle** is a walk without repeated vertices except that the first and last vertex are the same. The terms **path** and **cycle** also refer to specific graphs.

A graph is **connected** if between every two vertices there is a walk.
The **degree** of a vertex is the number of edges coming out of it.

In a graph, the sum of the degrees is twice the number of edges.