1. Let $X$ be a set with 1000 elements and $Y$ a set with 1001 elements.
   
   (a) How many functions from $X$ to $Y$?
   
   (b) How many functions from $Y$ to $X$?
   
   (c) How many one-to-one functions from $X$ to $Y$?
   
   (d) How many one-to-one functions from $Y$ to $X$?

2. Consider the universe of all 6-letter English words. In each case, state whether it is an equivalence relation or not, and if not, state one of the three properties it fails:
   
   (a) Define two words to be related if they end with the same letter
   
   (b) Define two words to be related if they both contain at least two vowels
   
   (c) Define two words to be related if the first one occurs before the second one in dictionary order.

3. Give a proof by contradiction that $\sqrt[3]{2}$ is irrational.

4. Recall that $2023 = 7 \times 17 \times 17$.
   
   (a) How many distinct factors does $2023$ have?
   
   (b) What is the largest factor of $2023^{2023}$ apart from itself?
   
   (c) What is the smallest factor of $2023^{2023}$ apart from 1?