1. Prove by induction that the sum of the first \( n \) odd numbers is \( n^2 \).

2. Prove by induction that \((2n)!\) is a multiple of \(2^n\) for all positive integers \( n \).

3. Prove by induction that
\[
1 + \frac{1}{5} + \frac{1}{25} + \ldots + \frac{1}{5^n} = \frac{5 - \frac{1}{5^n}}{4}.
\]

4. Prove by induction that \(2^{n+1} + 5^n\) is a multiple of 3 for all positive integers \( n \).

5. Find the problem with the following proof that all people are the same age:

We prove by induction on \( n \geq 1 \) that for any set \( S \) of \( n \) people all people in the set \( S \) are the same age. The base case is \( n = 1 \): there is only one person and they certainly are the same age as themselves. So assume the statement is true for \( n - 1 \), and test for \( n \). Let \( S \) be some set of \( n \) people. Pick any person \( A \) out of the set. By the IH, all the remaining people in \( S \) have the same age. So we just have to argue that \( A \) has the same age as everyone else.

If we start with \( S \) again and omit someone else say \( B \), by the IH we get that \( A \) is the same age as everyone else except \( B \). By repeating this with different choices of \( B \), we get that \( A \) is the same age as everyone else. So everybody in \( S \) has the same age.