1. Prove that the product of three consecutive integers is a multiple of 6.

2. State the converse and contrapositive of: If a rose is blue, then a pansy is red.

3. Prove by contradiction that if \( m^2 \) is odd then \( m \) is odd.

4. Prove by contradiction that any solution to the equation \( x^4 + x^2 + 1 = 0 \) is irrational.

5. Consider an 8 \( \times \) 8 checker-board, and a supply of dominoes which are the size of two adjacent squares of the board.

   (a) Show that one can use 32 dominoes to cover all the squares of the board.
   (b) Show that if one removes two diagonally opposite corners of the board, then it is not possible to use 31 dominoes to cover the remaining squares.

6. Find the problem with the following famous proof that 1 = 2:

   Let \( x = y \). Then \( x^2 = xy \), so that \( 2x^2 = x^2 + xy \). It follows that \( 2x^2 - 2xy = x^2 - xy \), and so \( 2(x^2 - xy) = x^2 - xy \). If we divide both sides through by \( x^2 - xy \), we get \( 2 = 1 \).

7. In each of the following, determine if the two statements are equivalent.

   (a) \( \forall x \, (p(x)) \) and \( \neg \exists y \, (\neg p(y)) \)
   (b) \( \exists x \, (\exists y \, (q(x, y))) \) and \( \exists y \, (\exists x \, (q(x, y))) \)
   (c) \( \forall x \, (s(x) \lor t(x)) \) and \( (\forall x \, (s(x))) \lor (\forall x \, (t(x))) \)