Assignment 1

1. This is based on the class activity. Determine with proofs:
   
   (a) The minimum number of colors needed to color the edges of $K_5$ so that no adjacent edges are the same color.
   
   (b) The minimum number of colors needed to color the edges of $K_5$ so that no triangle is monochromatic (all three edges the same color).
   
   (c) The maximum number of colors needed to color the edges of $K_5$ so that no triangle is rainbow (all three edges different color).

2. Speculate/discuss/guess how the answers would change in the previous question if 5 is replaced by 2024. (Proof not required.)

3. A graph with six vertices has a vertex of degree 1, one of degree 2, one of degree 3, one of degree 4 and one of degree 5. What is the degree of the remaining vertex? Explain.

4. Determine all connected graphs with the property that they do not contain an induced copy of the path on two edges.

5. For a graph $G$, let $IC(G)$ denote the number of subsets of the vertices that are independent. For example, $IC(P_3) = 5$ (the empty set, three singletons, and the pair of leaves).
   
   (a) Calculate $IC(K_{2024})$
   
   (b) Calculate $IC(K_{2024,2024})$
   
   (c) Calculate $IC(P_{2024})$

6. Show that if $G$ is a disconnected graph, then its complement $\overline{G}$ is connected.

7. The double wheel $D_n$ is constructed by taking the cycle $C_n$ and adding two new vertices $u$ and $v$ and making both of them adjacent to all the vertices of the cycle. There is no edge between $u$ and $v$.
   
   (a) What is the maximum size of an independent set in $D_n$?
   
   (b) What is the maximum size of a clique in $D_n$?
   
   (c) For which $n$ does $D_n$ have an Eulerian circuit?
   
   (c) Show that $D_n$ is decomposable into $P_3$ for $n$ even.

Due: Tuesday January 23