1. (a) Show that if $K_n$ has a $K_3$ decomposition, then $n$ is of the form $6k + 1$ or $6k + 3$ for $k$ an integer.
(b) Show that $K_7$ has a $K_3$ decomposition.

2. For a fixed graph $H$, define a graph as helplessly $H$-decomposable if no matter what choices one makes, one can always repeatedly remove copies of $H$ and use up all the edges of $G$. For example, $C_4$ is helplessly $P_3$-decomposable, but $P_5$ is not helplessly $P_3$-decomposable.
(a) Determine all helplessly $P_3$-decomposable graphs on at most six edges.
(b) Propose a general characterization about helplessly $P_3$-decomposable graphs.
(c) Prove your characterization.

3. (a) Determine all trees on 6 vertices, and for each, calculate the diameter, radius, average eccentricity, and Wiener index.
(b) Which tree(s) has the smallest value of the parameters? The largest?

4. Describe a graph that has exactly 2024 spanning trees.

5. (a) Draw a tournament of order 7 where exactly one vertex is a king.
(b) Draw a tournament of order 7 where every vertex is a king.
(c) Show that it is not possible for there to be exactly two kings.

Due: Tuesday February 6