

Assignment 2

1. Prove that if G is connected and has $\gamma(G) = n/2$ (where n is the number of vertices), then G is either a corona or C_4 .
2. Define a **2-dominating set** as a set S of vertices such that every vertex outside S is adjacent to at least 2 vertices in S , and let $\gamma_2(G)$ denote the minimum cardinality of a 2-dominating set of G .
 - (a) Compute γ_2 for the complete graph, complete bipartite graph, and the cycle.
 - (b) Show that if G has minimum degree at least 3, then $\gamma_2(G) \leq n/2$. (Hint: Show that there are two disjoint 2-dominating sets.)
3. An *art gallery with walls* is a polygon plus some non-intersecting chords called “walls” that join vertices. Each interior wall has a tiny opening called a doorway. A guard in a doorway can see everything in the two neighboring rooms, but a guard not in a doorway cannot see past a wall. Determine the minimum number t such that for every walled art gallery with n vertices, it is possible to place t guards such that every interior point is visible to some guard. (West: 6.3.13.)
4. Determine the domination, independent domination, and total domination numbers of $K_{2025} \square K_{2025}$.
5. Determine the chromatic number of
 - (a) $C_{2025} \square C_{2025}$
 - (b) $C_{2025} \times C_{2025}$
 - (c) $C_{2025} \boxtimes C_{2025}$

Due: Tuesday February 11