Math 9540 — Goddard — Spring25

Assignment 3

1. Determine all nonisomorphic circulants on 10 vertices.

2. The Clebsh graph S can be constructed by taking the hypercube Q_4 and adding a perfect matching pairing off antipodal vertices. (So the result is 5-regular.)

   (a) Show that S is triangle-free.

   (b) [Optional] Show that the Clebsh graph is isomorphic to the Cayley graph whose vertices are the elements of the finite field on 16 vertices, and two vertices x and y are adjacent if $x - y \in T$, where $T = \{ x^3 : x \neq 0 \}$.

   (c) Show that K_{16} can be decomposed into 3 copies of S.

3. Define $R(G_1, G_2, G_3)$ as the minimum N such that every coloring of the edges of K_N with $\{1, 2, 3\}$ produces a copy of G_i with all edges of color i for at least one i.

   (a) Calculate $R(K_{1,3}, K_{1,3}, K_{1,3})$.

   (b) Calculate $R(K_{1,3}, K_{1,3}, K_3)$.

   (c) Calculate $R(K_3, K_3, K_3)$.

4. Define $T(G)$ as the minimum N such that every 2-coloring of the edges of $K_{N,N,N}$ contains a monochromatic copy of G.

   (a) Calculate $T(P_3)$.

   (b) Calculate $T(C_4)$.

   (c) Show that $T(K_3)$ does not exist.

5. A tournament is an orientation of a complete graph. Show that there exists a tournament with $n!2^{-(n-1)}$ directed hamiltonian paths.

6. For fixed δ, use probabilistic methods to prove an upper bound on the 2-domination number of a graph with minimum degree δ.

Due: Tuesday February 25