

**Assignment 5**

**PART 1:** Answer questions on Canvas about talks.

**PART 2:**

1. (a) Show that the pair of rank axioms implies the pair of independent set axioms.  
(b) Show that the pair of independent set axioms implies the pair of rank axioms.
  
2. Recall that a graphical matroid is one that can arise from a graph.
  - (a) A uniform matroid is defined by taking the universe  $X$  and some integer  $k$  and defining as independent every set of size at most  $k$ . Determine which uniform matroids are graphical.
  - (b) A partition matroid is defined by taking a partition of the universe  $X$  into  $k$  classes and defining a set as independent if it contains at most one element from each class of the partition. Determine which partition matroids are graphical.
  
3. Show that a graph of diameter 2025 has at least 2026 distinct eigenvalues. (Hint: recall the Goodwill-Hunting and Cayley-Hamilton theorems.)

**Due: Wednesday April 9**