

## Introduction to Games

People love to play games. Games are fun! And while one can enjoy just playing them, the goal usually is to win. In this course we consider the techniques to analyze and systematically play various games. Sometimes we know how to play a game “perfectly”; sometimes we can just hope for a reasonable (computer) player.

### 1.1 Some Games

Here are five games to get you thinking. . .

**Finger.** Each player, say Anand and Betsy, holds up their hands clenched in a fist, no finger showing. Then they take turns. Let’s say Anand goes first. He chooses one of his hands and taps one of Betsy’s hands. Betsy then shows one finger on that hand. Betsy then does the same thing. In general, if Betsy taps with a hand that has  $x$  fingers showing, then Anand must increase the number of fingers showing by  $x$ ; if this brings the total to more than five on that hand, then Anand must put his hand behind his back for the duration. Tapping with a closed hand is equivalent to tapping with one finger. The last player with a hand out is the winner.

**Dueller.** Each player is dealt a hand with 5 cards. The duel starts with the players both playing a card face down. This is turned over and the worst card is discarded. The player whose card was defeated must then play a card to defeat the other player’s card. Play alternates until one player is unable to beat the other’s card and that player loses the duel.

The cards are compared as follows. If they are different colors, then Spades beats Hearts, Hearts beats Clubs, Clubs beats Diamonds, and Diamonds beats Spades. If they are the same color, then the higher denomination wins (A-K-Q-J-10-...-2). If they are the same color and the same denomination, then it is a tie and both cards are discarded; the players then restart the duel with a card face down.

**Nim.** There are three piles, one with 3 coins, one with 5 coins, and one with 7 coins. Players take turns to pick one pile and remove any number of coins from it (at least one coin, and may take all coins). The person who gets the last coin is the winner.

**BearingOff.** (The endgame of backgammon.) Consider a finish line, with six rows before the finish line. Each player places two counters on each row (a total of twelve counters). The goal is to be the first to get all of one’s counters across the finishing line. The player’s take turns to roll one dice. For a given roll, one may advance a

counter that value, provided that if there is another counter on a farther row and the move would take this counter across the finish line, then the value rolled is exact. For example, if there is one counter on row 5 and one on row 3, a roll of 3 can be moved by either  $5 \rightarrow 2$ , or  $3 \rightarrow \text{finish}$ , a roll of 4 can only be moved  $5 \rightarrow 1$ , and a roll of 6 can only be moved by  $5 \rightarrow \text{finish}$ .

**CodeBreaker.** There are two players. The CodeSetter creates a secret code of 4 colored pegs, each peg one of 6 colors. The CodeBreaker must infer the secret code by asking a series of questions. Each question is itself a candidate code, and the response is two integers called Black and White. The value of Black indicates in how many positions there is exact agreement, and the value of White indicates how many colors are correct but in the wrong position. In the actual game, the response is given in the form of little pegs.