Matrix Transforms
Domain, Codomain and Range

A function/transform/mapping has a domain and codomain: the domain specifies the possible inputs and the codomain specifies the possible outputs. The function maps each element of the domain to an image in the codomain.

The range is the set of all images: it is a subset of the codomain.
**Matrix Transform**

**Defn.** If the matrix $A$ is $m \times n$, then the matrix transform $x \mapsto Ax$ has domain $\mathbb{R}^n$, codomain $\mathbb{R}^m$, and range some subset of $\mathbb{R}^m$.

For example, if $A$ is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then the transform maps $(5, 3)$ to $(3, 5)$. 

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Some Matrix Transforms

- projections, such as $P = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

- shears, such as $S = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

- contractions/dilations, such as $C = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$

- rotations, such as $R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$
Summary

A transform maps each element of the domain to an image in the codomain. The range is the set of all images. A transform is onto if the range is the whole codomain, and one-to-one if every vector in the range is the image exactly once. Some matrix transforms have geometrical meaning.