Determinants: Definitions
A square matrix is invertible if and only if its determinant is not zero.
The 2 × 2 Case

\[
\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc
\]

We use both \textit{det} and vertical lines to indicate determinant.
**Permutation Matrices and Signs**

**Defn.** A *permutation matrix* is a square matrix that contains only 0’s and 1’s with exactly one 1 in each row and column.

**Defn.** The *sign* of a generalized permutation matrix is $(-1)^k$, where $k$ is the number of row interchanges needed to change the matrix to be diagonal.
**Definition of Determinant**

**Defn.** The determinant of matrix $A$: construct all possible permutation matrices $P$ and for each, multiply relevant entries of $A$ together then by sign of $P$, and sum the results.

For example:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$ has perm matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ & $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

The former has positive sign with transversal $ad$; the latter has negative sign with transversal $bc$. So we get $ad - bc$. 

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Some Consequences

**Fact.** If matrix $A$ has an all-zero row or column, then $\det A = 0$.

**Fact.** The determinant of a triangular matrix is the product of the diagonal entries.

In particular, the determinant of the identity matrix $I$ is 1.
A permutation matrix is square matrix with exactly one 1 in each row and column and all other entries zero. Its sign is \((-1)^k\), where \(k\) is number of row interchanges needed to change it to diagonal.

The determinant of a matrix is defined by: construct all possible permutation matrices and for each, multiply the relevant entries together then by the sign, and then sum the results.
The determinant of \( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \) is \( ad - bc \). The determinant of a triangular matrix is the product of the diagonal entries. A square matrix is invertible if and only if its determinant is not zero.