Linear Transforms
**Defn.** A **linear transform** $T$ is a function from one vector space to another vector space that interacts nicely with addition and scalar multiplication.

That is, for all vectors $u$ and $v$ in the domain and all reals $c$:

1. $T(u + v) = T(u) + T(v)$, and
2. $T(cu) = cT(u)$. 

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Every Matrix Transform is a Linear Transform

Fact. Every matrix transform is a linear transform.
The Kernel of a Linear Transform

**Defn.** The null space of a linear transform is the set of vectors that are mapped to 0; it is often called the **kernel**.

For example, differentiation is a linear transform from the polynomial space \( \mathbb{P}_n \) to the polynomial space \( \mathbb{P}_{n-1} \). Its kernel is the set of all constants.
Facts about Linear Transforms

Fact. For any linear transform:
1) The kernel is a subspace of the domain.
2) The range is a subspace of the codomain.
A linear transform is a function from a vector space to a vector space that interacts nicely with addition and scalar multiplication. Every matrix transform is a linear transform.

The null space or kernel of a linear transform is the set of all vectors mapped to 0. The kernel is a subspace of the domain while the range is a subspace of the codomain.