
Exercises for Chapter 1

A1. Solve the following systems.

$$\begin{array}{rcl} s + t = 5 & & x - y + z = 8 \\ 2s + t = 1 & & 2x - y = 4 \\ & & 3y = -6 \end{array}$$

A2. Consider this matrix

$$B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- (a) Is matrix B in echelon form? Explain.
- (b) Is matrix B in reduced row echelon form? Explain.

A3. Describe geometrically the solution to the linear equation $x + y + z = 1$.

A4. For what value(s) of β is the system $x + \beta y = 1$ and $x - y = \beta$ consistent?

A5. What's the maximum possible number of nonzero entries in a 3×3 matrix in reduced row echelon form? Justify your answer.

A6. Find the solution set of the systems given by the following augmented matrices. (Assume variables are x_1, x_2, \dots)

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 4 & -2 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right] \qquad \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 2 & -12 \\ 2 & 0 & 2 & 0 & 3 & 3 \end{array} \right]$$

A7. Consider n numbers x_1, x_2, \dots, x_n laid out on a circle and some value α . Consider the requirement that every number equals α times the sum of its two neighbors. For example, if α were zero, this would force all the numbers to be zero.

- (a) Show that, no matter what α is, the system has a solution.
- (b) Show that if $\alpha = \frac{1}{2}$, then the system has a nontrivial solution.
- (c) Show that if $\alpha = -\frac{1}{2}$, then there is a nontrivial solution if and only if n is even.

Some Solutions

- A1. $s = -4, t = 9$
 $x = 1, y = -2, z = 5$
- A2. (a) Yes: we have zeroes below the first nonzero in each row
(b) Yes: the pivots are 1 and have zeroes above them
- A3. A plane.
- A4. We can solve for x and y except when $\beta = -1$.
- A5. Four. In the case that there are pivots in first two columns only.
- A6. $x_1 = 2/5 + x_3, x_2 = -7/10 + x_3$
 $x_4 = 42 - x_3, x_5 = -27$
- A7. (a) All x_i zero provides a solution
(b) All x_i the same value
(c) Alternate +1 and -1. (But why impossible if n odd?)