Exercises for Chapter 6

F1. Consider the matrix

$$G = \begin{bmatrix} 1 & 2 & 0 & -3 \\ 2 & 7 & 1 & 7 \\ -2 & -1 & 1 & 19 \end{bmatrix}$$

(a) Give the dimension and a basis of the column space of G.

(b) Give the dimension and a basis of the null space of G.

(c) Give the dimension and a basis of the row space of G.

F2. Assume D is a 6×8 matrix of rank 5. What are the dimensions of $Nul\,D$, $Row\,D$, and $Col\,D$?

F3. If A and B are matrices, show that

(a) the column space of the product AB is contained within the column space of A.

(b) the rank of AB is at most the rank of A.

F4. Change of basis. What matrix:

(a) transforms $(1,0)$ to $(3,-4)$ and $(0,1)$ to $(1,2)$?

(b) transforms $(3,-4)$ to $(1,0)$ and $(1,2)$ to $(0,1)$?

(c) transforms $(3,-4)$ to $(1,0)$ and $(-6,8)$ to $(0,1)$?

Some Solutions

F1. (a) 2. Basis: $(1, 2, -2)$, $(2, 7, -1)$
(b) 2. Basis: $(2/3, -1/3, 1, 0)$, $(35/3, -13/3, 0, 1)$
(c) 2. Basis: $(1, 0, -2/3, -35/3)$, $(0, 1, 1/3, 13/3)$

F2. $Nul\, D$ has dimension $8 - 5 = 3$, $Row\, D$ has dimension 5, and so does $Col\, D$.

F3. (a) By definition of matrix multiplication, each column of AB is a linear combination of the columns of A. That is, each column of AB is in the column space of A.
(b) The column space of AB is contained within the column space of A, and therefore has dimension at most that of the column space of A.

F4. (a) $\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$
(b) $\frac{1}{10}\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$
(c) Does not exist