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A MODEL FOR VEHICULAR INTERACTIONS EXTRACTED FROM REAL-WORLD TRAFFIC DATA

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ABSTRACT

With the advent of connected and automated vehicles, naturalistic traffic modeling is becoming increasingly important for the purposes of congestion control via cooperative driving and intelligent traffic management. The focus of this paper is to develop a data-driven traffic interaction model which can help in the development of multi-agent automated driving control schemes that mimic and blend with other human-driven vehicles. To that end, a probabilistic data analysis approach is used to derive an energy function that describes interactions between vehicles on highways. These interactions arise from the psychophysics of humans driving the vehicles. The analysis suggests the existence of a simple interaction law between human-driven vehicles that is based on the expected time it takes for the vehicles to collide. The approach taken in this paper helps in analyzing probable states that the individual vehicles in the traffic would have, thereby facilitating the development of intelligent traffic management tools that account for individual vehicle states.

1 INTRODUCTION

Modeling vehicular traffic is essential to understand driving patterns and assessing and improving existing roadway infrastructures. Over the past thirty years many models have been proposed for modeling the movements and dynamics of vehicles in traffic. The models are often grouped as *microscopic*, *macroscopic*, and *cellular automata* models. Microscopic models focus on individual agents (vehicles, pedestrians) and study how these agents behave and interact with other agents in traffic. Here, similar to particle systems, the behavior of the agents is modeled to arise from the resultant of different force components. In contrast, macroscopic models focus on the behavior of the traffic as a whole, where the agents behave as "thinking fluids". Here, the emphasis is on the characteristics of the traffic flow rather than on individual variability. These concepts are explored in detail in [1]. Use of cellular automata is another widely used modeling construct to describe traffic flow in homogenous and mixed setting [2]. In such approach, the state space is discretized into a regular grid of cells, and at each time step, each agent moves according to a set of rules.

Despite the successful application of the aforementioned approaches to traffic modeling, the majority of the existing approaches follow a simulation-driven paradigm, where a model of inter-vehicular interactions is hypothesized and verified in simulation. As such, the resulting behavior can deviate from actual human-driver behavior rendering these approaches problematic for actual use in advanced driver assistance systems and self-driving solutions. To address this issue, in this paper, we seek to directly *infer* from real world traffic data a general quantitative law for characterizing interactions between vehicles. Our current focus is on how human drivers interact on highways, thereby facilitating the development of motion planning and controls algorithms for autonomous highway navigation such as nonlinear model predictive control planners (NMPC) [3]–[5].

In particular, in this work, we analyze the natural vehicle-vehicle interactions that exist in a publicly available highway dataset and seek to derive an interaction law that can be seamlessly integrated into existing approaches for predictive motion planning and control of (semi-) autonomous vehicles. While inverse reinforcement learning approaches provide a natural way to infer objective functions from human data [6], [7], such approaches are typically tied to a specific task and cannot easily generalize to different conditions. Instead, following the recent work in [8], we are going to employ a *probabilistic analysis approach*. The main idea, here, is to compare the distribution of observed states in the data to a purely random distribution. The output of this analysis is an interaction law expressed as an *energy function* which can be used to resolve collisions between individual agents. In [8], it was shown that the interaction energy between two pedestrians follows an inverse power-law relationship with their expected time to collision. This has now found applications in the fields of character animation and robotic navigation. In this paper, we show that time to collision is again a sufficient descriptor of inter-vehicular interactions. However, the energy function that describes interactions between vehicles is of a different form than the pedestrian interaction energy.

Overall this paper makes the following contributions:

1. We show how traffic data can be analyzed using a probabilistic analysis approach based on tools from statistical mechanics.

2. We investigate different variables that can be used to describe interactions between vehicles from traffic data, including variables local in time such as distance, and anticipatory variables such as minimal predicted distance and time to collision.

3. By applying our analysis technique to a publicly available traffic dataset, we infer a *general quantitative* law for describing inter-vehicular interactions on highways; our analysis shows that the strength of the interaction between two vehicles increases exponentially as the expected time that it takes for the two vehicles to collide decreases.

4. The inferred interaction law can be incorporated into existing predictive planners such as NMPCs as part of their optimization function, enabling natural resolution of collisions between road vehicles at different levels of automation.

We argue that automated driving systems (ADS) that incorporate this naturalistic interaction laws in their motion planning and control schemes can facilitate the integration of ADS into the existing roadways with smooth interactions with other human-driven vehicles.

The rest of the paper is organized as follows: Section 2 describes the dataset we work with. We detail the derivation of the interaction energy in Section 3. In Section 4 we discuss alternative descriptors, and in Section 5, we analyze the interaction law further with the time to collision as the descriptor. We summarize the conclusions and directions for future work in Section 6.

2 DATASET

The traffic dataset used in this study comes from US DOTs Next Generation Simulation (NGSIM) project [9]. In that project, vehicle trajectory information was collected on highways I-80 and US-101 in California and on city streets Lankershim Blvd in California and Peachtree street in Atlanta, Georgia. At every location, the dataset is further sorted based on the traffic density into uncongested, buildup to congestion and congested datasets. This sorting is done by recording data during specific times of the day when the traffic is sparsely (uncongested), moderately (buildup congestion), highly (congested) dense. TABLE 1 gives more information about these datasets.

The NGSIM dataset includes several useful granular information on the observed traffic. The information used for this work is vehicle ID, frame ID, local X coordinate, local Y coordinate, vehicle length, vehicle width, vehicle velocity, vehicle acceleration, lane ID and vehicle class which indicates whether the vehicle is a car, bike or truck/bus. As, the lateral and longitudinal velocities of the vehicles are not available in the dataset, we simply inferred them from the longitudinal and lateral positions and their timestamps.

TABLE 1: INFORMATION ON THE DATASET

Dataset	Length	No. of lanes	Flow
I-80	503 m	6	Uni-directional
US-101	640 m	6	Uni-directional
Lankershim	500 m	8	Bi-directional
Peachtree	609 m	5	Bi-directional

The available dataset includes vehicle trajectories on highways and roads which are more than 500 meters long and are up to 6 lanes wide in one direction. Figure 1 shows the layout of highway I-80 along with two vehicles. Two vehicles are said to be interacting when the drivers of the respective vehicles can perceive each other and modify their state based on the state of the other vehicle. As shown in the figure, vehicles A and B are too far from each other for the drivers to perceive each other and interact. So, it is necessary to remove such pairs from data analysis in order to make sure it doesn't statistically suppress the nature of the real interactions.

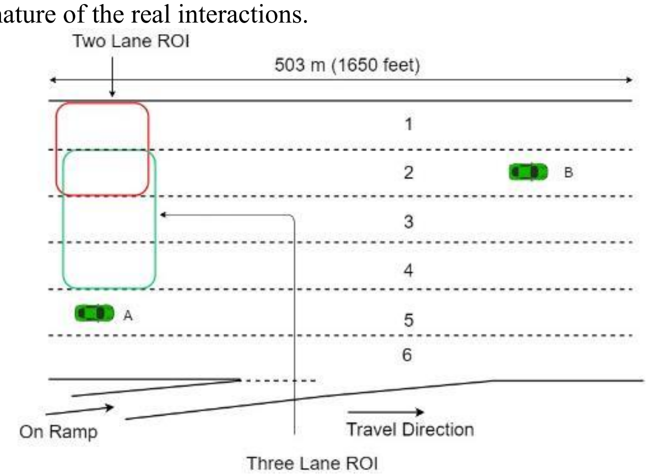

FIGURE 1: LAYOUT OF FREEWAY I-80. RED BOX INDICATES 2 LANE REGION OF INTEREST (ROI) AND GREEN BOX INDICATES 3 LANE ROIs.

To solve this problem, a region of interest (ROI) approach was taken to remove vehicles which were too far from each other to interact. It is only when both the vehicles are inside a specific ROI that they are said to be interacting and are considered for data analysis. The vehicles outside of ROI are ignored all together. The length and width of the ROI were determined taking into account the average speed of the vehicles and the field of view of a human driver in dense traffic. A ROI on the highway boundaries of 150m long and 2 lanes wide was taken which is shown in the red box in Figure 1. For inner lanes, the ROI width was expanded to 3 lanes, shown by a green bounding box in Figure 1. The green bounding box is longitudinally shifted just for representation purposes. The lane which connects to the on-ramps and exits on the freeway was ignored for the analysis because of the lack of vehicles travelling through that lane as compared to other five lanes shown in Figure 1. Based on possible permutations from the remaining lanes, four ROIs were created for every traffic density condition on freeway I-80 and US-101. The ROIs were later combined for a generalized solution for specific traffic density conditions on specific freeways. Combining ROIs just helps in creating a larger statistical set. The region of interest can't be made too small to make sure that enough pairs of vehicles are present for the ensuing statistical data analysis.

3 DERIVATION OF THE INTERACTION ENERGY FUNCTION

In this section, the approach used to derive the inter-vehicular interaction model is discussed. The main idea, here, is to compare the distribution of observed states in the data to a purely random distribution. So, if a state happens way less frequently than random, this implies a large energy/cost that prevents such a state from occurring often. If a state happens as frequently as random, then there is no interaction, and, hence, no energy is associated with this state. As in [8], we can formalize this intuition using the pair distribution function, g, from statistical mechanics, which is robust to noisy and incomplete data [8].

Formally, in our domain, g describes the extent to which different configurations between pairs of vehicles are made unlikely by their mutual interaction (see Section 3.1). In situations where the intensive properties of the system do not vary strongly with time, this statistical suppression can be expressed in terms of an "interaction energy" as explained in Section 3.2.

3.1 Pair Distribution Function

Assuming a descriptor variable k is used to describe the interaction between a pair of vehicles, let $g(k)$ denote its pair distribution function. We discuss some options for the choice of k in Section 4. As in the typical condensed matter setting, we define the pair distribution function as $g(k) = \frac{P(k)}{P_{ni}(k)}$, where $P(k)$ is the probability density function for descriptor k between pairs of *interacting* vehicles in the dataset, and $P_{ni}(k)$ is the probability density function for the descriptor k between pairs of *non-interacting* (random) vehicles.

Given the discrete nature of the NGSIM dataset, the probability density functions are approximated using normalized histograms. In this setting, we define two vehicles as interacting if they are in the same region at the same time. However, for non-interacting pairs the probability density function $P_{ni}(k)$ needs to be approximated, because it is impossible to know beforehand all the non-interacting pairs given the dynamic nature of highway traffic. One way to approximate this is to sample pairs of vehicles that are not simultaneously present in the scene, e.g., by randomly permuting the time information of each vehicle while keeping its position and velocity information. This scrambling of time would make two vehicles in a scene unrelated and non-interacting.

Note that due to the way $g(k)$ is defined, we expect that vehicles do not influence each other for large descriptor values, i.e., $\lim_{k\to\infty} g(k) = 1$. Therefore, the pair distribution function leads to a plateauing behavior when pairs of vehicles start exhibiting normal behavior. Consider, for example, using the Euclidean distance between two vehicles as a descriptor. Most of the drivers prefer to maintain a certain separation, x, from nearby vehicles, which means that beyond x, two vehicles are not interacting with each other. In contrast, it's highly unlikely to observe two vehicles that have a very small separation value, i.e. $g(x) \ll 1$. Such small x's result in strong interactions, and the pair distribution function allows us to highlight such statistically suppressed configurations.

3.2 Interaction Energy Function

The datasets are split into three 15-minute windows where the traffic density is fairly constant (low, medium, and high). In addition, within each time window, other traffic parameters such as average speed remain fairly constant. This statistical equilibrium allows us to use a Boltzmann-like relation to describe a relation between the pair distribution function, $g(k)$, and the interaction energy, $E(k)$ [10]**.** Formally, the interaction energy can be expressed in terms of $g(k)$ as:

$$E(k) \propto \log[1/g(k)] \tag{1}$$

A pseudocode has been shown below which explains the computation of interaction energy from the vehicle states.

ALGORITHM 1: COMPUTATION OF INTERACTION ENERGY

```
INT =[], NINT=[]  // two lists that store descriptor values for interacting
                  //  and non-interacting pairs of vehicles
Repeat for every time frame
  Repeat for every pair of vehicles
      INT.append(Compute descriptor)
      Create time-scramble pair // For each vehicle randomly select
                                // one of its states ignoring time
      NINT.append(Compute descriptor for the random pair)
  end
end
f_int  ← histogram(INT)    // Compute histogram for interacting pairs
f_nint ← histogram(NINT)   // Compute histogram for time-scrambled pairs
g ← f_int / f_nint         // Compute PDF
E ← log(1/g)               // Compute interaction energy
```

3.3 Considerations of Vehicle Geometry

Depending on the descriptor selected, a specific vehicle geometry may need to be defined. If we take the Euclidean distance between a pair of vehicles as the descriptor. The coordinates of vehicles in each time frame is enough information to create a pair distribution function. However, for time to collision that is not the case. In order to compute time to collision between a pair of vehicles, we need to define a geometric shape for them and then compute time to collision accordingly. There are multiple unique ways in which a vehicle geometry can be described. For smooth description, a common choice is ellipse [11] or an approximation of it is used instead [12]. The problem with using an ellipse to describe a vehicle is that the computation of the time to collision (TTC) becomes complex [13]. This process can be avoided by using simpler geometries, with the most simple yet effective one being a circular one [14]. However, using the prescribed circle of a vehicle to approximate its geometry can be too conservative and lead to inaccurate time-to-collision computations. As such we use a collection of overlapping discs to approximate the shape of any vehicle as shown in Figure 2 [12].

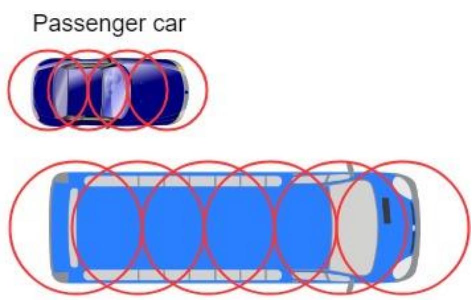

FIGURE 2: VEHICLE GEOMETRY LAYOUT FOR TTC COMPUTATION [12].

4 CHOICE OF DESCRIPTORS

In this section, we investigate three different descriptors, k, for the pair distribution $g(k)$:

1. Euclidean distance
2. Minimal predicted distance (MPD)
3. Time to collision (TTC)

The Euclidean distance just captures the relative location of two vehicles, whereas MPD and TTC capture both relative location and relative velocity between the pair of vehicles. In order to facilitate the appropriate choice of a descriptor k, it is necessary to verify whether such a descriptor is *sufficient* to generalize well to different cases. This means that the shape of the pair distribution function $g(k)$, and subsequently the shape of the interaction energy function $E(k)$, should depend only on k and be independent of all other variables like the relative velocity between two vehicles. This is going to be analyzed further for each of the three aforementioned descriptors. Euclidean distance and MPD do not require specification of vehicle geometries for their computation. However, to compute TTC, the vehicle geometry described in Section 3.3 was used. Specifically, we use analytical formulae for the TTC between each pair of discs belonging to different vehicles in the specific region of interest (ROI) and take the smallest value as the TTC between two vehicles under consideration (See Section 4.3).

The pair distribution function results shown in this section are for a specific ROI on freeway I-80 during the congestion buildup phase and on lanes 3,4,5. The ROI is 150 meters long. The bin size used to generate the corresponding histograms for the pair distribution function is 0.1 seconds/0.15 meters.

4.1 Euclidean Distance

The Euclidean distance d between a pair of vehicles can be computed based on the (x,y) coordinates of the vehicles, which are readily available in the NGSIM dataset. The corresponding pair distribution function is shown in Figure 3. The peak at about 4 meters indicates a local maximum in the probability density function before plateauing close to the value of one. This is predominantly caused by large portion of the drivers maintaining the distance of 4 meters to the vehicle on either side. The reason for a distacne of 4 meters is that due to traffic density and standard lane width of US freeways which is 3.7 meters. In high traffic density conditions, large number of vehicle travel close to each other in parallel lanes which causes large number of pairs of vehicles to have a distance of 4 meters separating them. To verify whether Euclidean distance is a sufficient descriptor of inter-vehicular interactions, we binned the data based on how fast two vehicles approach each other. Ideally, we would expect the shape of the pair distribution functions to be independent of the relative velocity between vehicles.

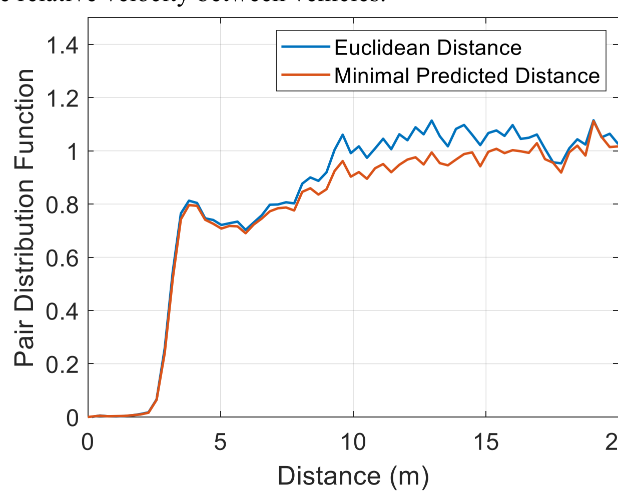

FIGURE 3: PAIR DISTRIBUTION FUNCTION FOR EUCLIDEAN DISTANCE AND MINIMAL PREDICTED DISTANCE

Figure 4 shows the corresponding plots obtained by splitting the data into three clusters. The range of relative velocities were chosen to make sure there is fairly equal number of pairs of vehicles per cluster. All three pair distribution functions have a local maximum just like the overall pair distribution function. At low relative velocity, though, the pair distribution function is significantly different from the other two curves. Euclidean distance is therefore not a good descriptor of microscopic traffic interaction as it only accounts for the relative position of the vehicles ignoring their relative velocity.

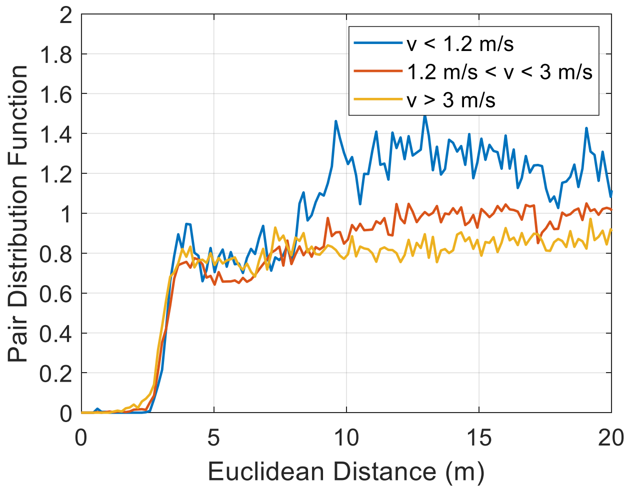

FIGURE 4: PAIR DISTRIBUTION FUNCTION FOR EUCLIDEAN DISTANCE FOR DIFFERENT RELATIVE VELOCITIES

4.2 Minimal Predicted Distance

As defined in [15], the minimal predicted distance (MPD) denotes the closest distance that two vehicles can attain at any time assuming an extrapolation of their trajectories based on their current velocities. Like Euclidean distance, MPD can be computed without specifying vehicle geometry. Formally, given two vehicles, the MPD is computed as

$$MPD = \min_{t\geq 0} ||(\mathbf{x_1} - \mathbf{x_2}) + (\mathbf{v_1} - \mathbf{v_2}) * t)|| \qquad (3)$$

Here $\mathbf{x_1}$,$\mathbf{v_1}$ is the position and velocity vector of vehicle 1 respectively and $\mathbf{x_2}$,$\mathbf{v_2}$ is the position and velocity vector of vehicle 2 respectively. t is the time to minimum separation.

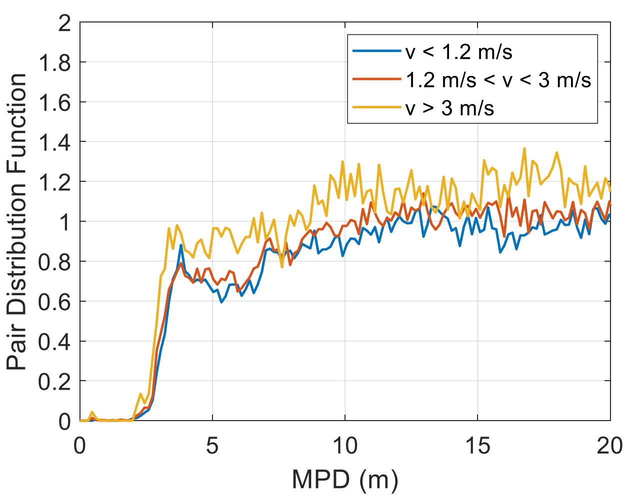

FIGURE 5: PAIR DISTRIBUTION FUNCTION FOR MINIMAL PREDICTED DISTANCE FOR THREE DIFFERENT RELATIVE VELOCITIES

The corresponding pair distribution function has a local peak when MPD is about 4 meters as shown in Figure 3. The pair distribution function based on MPD is not completely independent of the relative velocity. This is shown in Figure 5 where the pair distribution function for higher relative velocity pairs shows small signs of separation from the other two plots. Having such separation indicates that there is different interaction energy equation for different relative velocity between vehicles. Therefore, minimal predicted distance may not always be ideal to characterize traffic interactions in general situations with varying relative velocities. However, there is a significant improvement over the Euclidean distance plots (see Figure 4), which demonstrates the importance of anticipation (prediction) for describing interactions between human drivers.

4.3 Time to collision (TTC)

The time to collision (TTC) is the time required for two vehicles to collide if they continue at their present speed and maintain their current trajectories. The assumption here is that the vehicles have a linear motion and their velocities do not change from their present value. In this case, time to collision, is understood to be a function of the relative displacement and relative velocity of the two vehicles, computed as:

$$TTC = \min(t: t \geq 0, \|\mathbf{x_{12}} + \mathbf{v_{12}}t\ \| \leq r_{12}\ ) \qquad (4)$$

Here $\mathbf{x_{12}}$ is the relative position vector of the vehicle 1 with respect to vehicle 2. $\mathbf{v_{12}}$ is the relative velocity vector between vehicle 1 and vehicle 2, r_{12} is the combined radius of the circles describing vehicle 1 and 2, and t is anytime that satisfies the criteria shown in equation 4. As each vehicle is represented as a collection of circles shown in Figure 2, the time to collision is defined as the minimum TTC between all the possible combination of pairs of circles between the two vehicles.

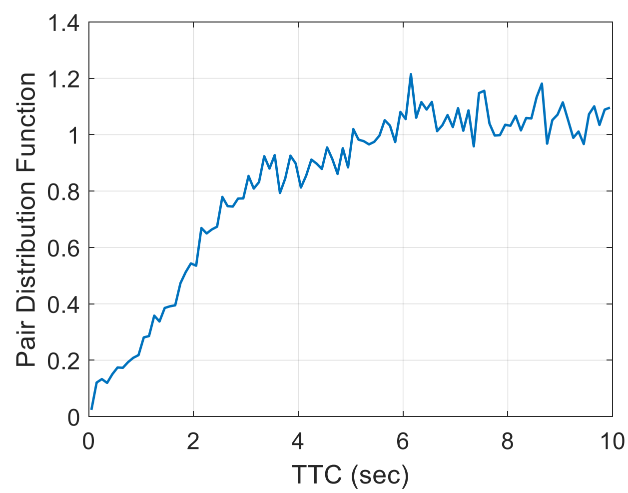

FIGURE 6: PAIR DISTRIBUTION FUNCTION FOR TIME TO COLLISION

The corresponding pair distribution function shown in Figure 6 is much smoother than the distance-based and MPD-based one. Importantly, the pair distribution functions generated for different relative velocities collapse onto each other as shown in Figure 7.

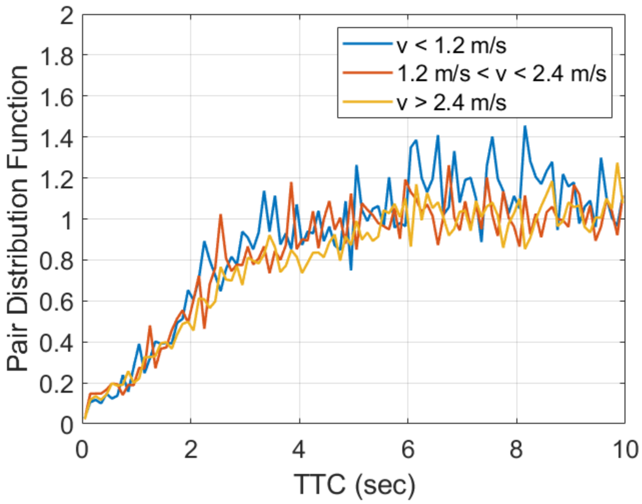

FIGURE 7: PAIR DISTRIBUTION FUNCTION FOR TIME TO COLLISION FOR DIFFERENT RELATIVE VELOCITIES

Even when binned by other parameters such as traffic density (see Figure 8), there is no significant separation between the different curves. This goes to show that time to collision is an ideal descriptor of traffic interaction. It takes into account both relative displacement and velocity which makes it ideal for our domain.

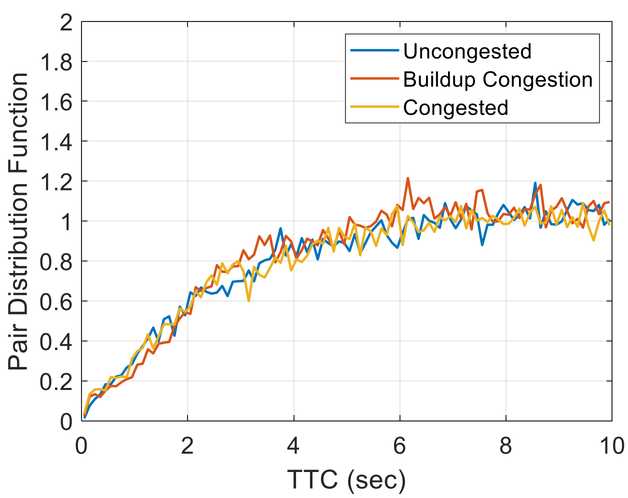

FIGURE 8: PAIR DISTRIBUTION FUNCTION FOR DIFFERENT DENSITY TRAFFIC CONDITIONS.

5 INTERVEHICULAR INTERACTION LAW

Having identified the time to collision as a sufficient descriptor of inter-vehicular interactions, in this section, we derive the corresponding energy law governing the collision avoidance behavior of an interacting pair of vehicles.

5.1 Region of Interest Accumulation

In the previous sections, the reported pair distribution functions were obtained by focusing on a specific ROI during the buildup congestion phase on freeway I-80. Before diving into the computation of interaction energy from such pair distribution functions, we need to ensure the availability of a sufficient amount of data to draw statistically significant conclusions. Therefore, here all ROIs within a given traffic density condition will be considered. For example, for freeway I-80 for congestion buildup phase we would accumulate the following ROIs: lane 1,2; lane 1,2,3; lane 2,3,4; lane 3,4,5; and lane 4,5 as explained in Section 2. Figure 9, inset, shows the corresponding pair distribution function. The addition of more data to the interacting and non-interacting histograms for the computation of pair distribution function made the function denser and more descriptive as compared to individual region of interest shown in Figure 6.

5.2 Interaction Energy

We can now use Equation (1) to estimate the interaction energy from the pair distribution functions. Figure 9 plots the interaction energy for vehicles on I-80 during buildup congestion. The red solid line shows the fit to the data, revealing that the energy decays exponentially as a function of time to collision:

$$E = a * \exp(-b * TTC) \qquad (5)$$

Here, a is a scaling constant that sets the units for the energy, and hence $E(TTC) \propto \exp(-b * TTC)$. For small TTC values (less than $\sim$0.2s), the energy seen in the data saturates to a maximum value, likely as a consequence of finite human reaction times. On the other hand, the observed interaction energy vanishes for sufficiently large TTC values, indicating that, during buildup congestion, human drivers tend to ignore collisions that will take place after $\sim$5.2s.

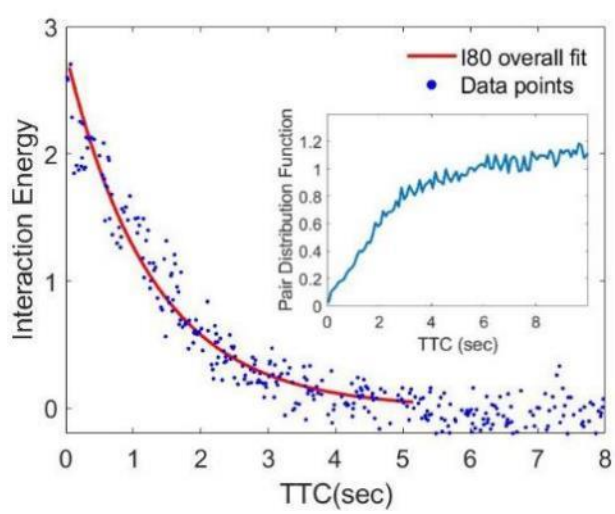

FIGURE 9: DATA FITTING FOR INTERACTION ENERGY FUNCTION FOR ACCUMULATED REGION OF INTEREST ON I-80 DURING BUILDUP CONGESTION. INSET SHOWS THE PAIR DISTRIBUTION FUNCTION FOR SAME DATASET

Importantly, despite their large qualitative differences, similar exponential fits were obtained for the interaction energy on different freeways and different traffic conditions as shown in TABLE 2. Here, we estimated $g(TTC)$, and subsequently $E(TTC)$, using histogram bins of 0.01s. In addition, we restricted our analysis to the range of time to collision values for which interaction energy values are significant. We note that for visual clarity in Figures 9 and 10, only 50% of the data points are shown.

TABLE 2: COEFFICIENTS FOR DATA FIT FOR MULTIPLE TRAFFIC CONDITIONS AND HIGHWAYS

Freeway	Traffic Conditions	*b*	TTC low(sec)	TTC high(sec)
I-80	Uncongested	-0.78	0.2	5.6
	Buildup Congestion	-0.77	0.2	5.2
	Congested	-0.88	0.2	4
US-101	Uncongested	-0.78	0.2	6.2
	Buildup Congestion	-0.56	0.4	6
	Congested	-0.63	0.4	6

As can be seen from TABLE 2, the traffic density influences only slightly the coefficient b of the exponential decay energy functions for I-80 and US-101. In fact, on I-80, the interaction energy for both uncongested and buildup congestion, can be well modeled with an exponential function having the same coefficient, and the same applies to the uncongested US-101 dataset. This suggests that the inferred interaction law can account for variation in traffic density without requiring any additional energy terms. It's also worth noting that during congestion on I-80, drivers consider a shorter interaction horizon as compared to sparse traffic conditions and/or drivers on US-101. Figure 10 shows the interaction energy for the I-80 dataset after having combined all the ROIs and densities together.

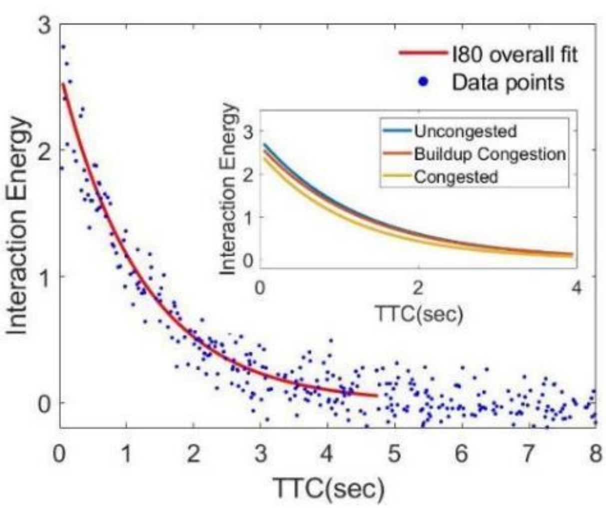

FIGURE 10: DATA FITTING FOR I-80 DATASET COMBINING ALL REGIONS OF INTEREST AND TRAFFIC DENSITIES

TABLE 3 highlights the corresponding energy fit along with the one obtained for the overall US-101 dataset. As can be observed from the table, in both datasets, the interaction energy between vehicles decays exponentially as their expected time to collision increases, with vehicles on US-101 exhibiting slightly stronger interactions than vehicles on freeway I-80.

TABLE 3: COEFFICIENT FOR DATA FIT FOR MULTIPLE HIGHWAYS

Freeway	*b*	TTC low(sec)	TTC high(sec)
I-80	-0.81	0.2	4.8
US-101	-0.66	0.5	6

6 CONCLUSION

In this paper, the nature of the interaction between human driven vehicles on highways were analyzed using tools from statistical mechanics. Our analysis indicates that interactions between vehicles are governed by a single energy function that is anticipatory in nature. Even though the coefficients of the interaction function can vary between different densities and highways, overall the energy increases exponentially as the anticipated time to collision between two vehicles decreases. We believe that the inferred energy law can provide a much more natural alternative over interaction potentials obtained via simulation-driven techniques [5] or probabilistic constraint evaluations [11].

The interaction energy law we obtained in this paper can form a naturalistic cost function of a predictive planner for automated driving system that can be used for smoothly integrating automated vehicles among a traffic dominated by other human driven vehicles, as is likely to be the case as the former get introduced on public roadways. Some of our preliminary results in this direction look quite promising. We would also like to start investigating interactions laws between human-driven vehicles and pedestrians, which will lead to a much more generalized framework for predictive, agent-based traffic simulation, where the behaviors of the agents are governed by interaction energy functions.

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