APPENDICES

Appendix A. Adapting a TCP Throughput Model for DCA

In this Appendix we overview the TCP throughput model and show how we adapt it for our DCA analysis. The model is based on the concept of a round. A round starts with the back-to-back transmission of W packets where W is the current size of the TCP *cwnd*. No other packets are sent until the first ACK is received for one of these W packets. This ACK reception marks the end of the current round and the beginning of the next. During a round, the window size increases by 1/b where b is the number of packets that are acknowledged by a received ACK. There are three possibilities associated with a round.

- Packet loss can be detected by the arrival of "triple-duplicate acknowledgements" (this event is referred to as a TD loss indication).
- Packet loss can be detected by a TCP timeout (this event is referred to as a TO loss indication)
- The round is impacted by a maximum window value.

A TD period (i.e., a TDP) is defined as the period of time between consecutive TD loss indications. The analysis assumes that the evolution of the congestion window size consists of a sequence of TD periods (Z_i^{TD}) and a sequence of TO's (Z_i^{TO}). We define the time period S_i as:

$$S_i = Z_i^{TD} + Z_i^{TO}$$

Define M_i to be the number of packets sent during S_i. The average TCP throughput (*B*) achieved during the time period is defined as:

$$B = \frac{E[M]}{E[S]}$$

After some analysis, this can be written as:

$$B = \frac{E[Y] + Q * E[R]}{E[A] + Q * E[Z^{TO}]}$$

Where *Y* is the number of packets sent during the portion of S_i associated with TD events (denoted as *A*). *R* denotes the number of packets sent during timeout sequence Z_i^{TO}. *Q* is the probability that a loss indication ending a TDP is a TO.

We want to modify the model to approximate the effect of a hypothetical DCA algorithm. Assuming a DCA congestion reaction is equivalent to the recovery associated with a TD loss indication, the effect of DCA will generally be to increase the number of TD indications (due to the congestion decisions) and to decrease the number of timeouts (since the actual number of packet losses will be reduced). The approach takes advantage of the fact that the portion of the model that accounts for TD loss indications (*E[Y]* and *E[A])* are independent of timeout events. Similarly, the *Q*, *E[R]* and *E[Z]* components of the model account only for the impact of timeouts to the throughput and are independent of the *E[Y]* and *E[A]* components. We modify the model to compute the *E[Y]* and *E[A]* based on a probability of TD indications that will be inflated by some amount due to DCA reactions *(pTD)*. We compute the *Q, E[R] and E[Z]* based on a packet loss rate that is reflective of the actual packet loss rate the connection experiences (*pTO*).

Below we list the results based on an assumption that the receiver acknowledges every two segments that arrive. There are two cases depending on if the connection is bound by the maximum window (i.e., *W(p)*<*W(max)*). The *W(p)* is the average window value based on the packet loss probability. For our purposes, the *pTD* will always be larger than the *pTO*. Therefore, we compute the *W(p)* based on the *pTD* probability. The *B* value is the average duration of the retransmit timeout amount.

If $W(p) < W(\max)$:

$$W(p) = 2/3 + \sqrt{\frac{4*(1-p)}{3p} + 4/9}$$ based on $p=pTD$

$$f(W(pTD), pTD, pTO, R, B) = \frac{\frac{1-pTD}{pTD} + W(pTD) + \frac{Q(pTO, W(pTD))}{1-pTO}}{R(W(pTD)+1) + \frac{Q(pTO, W(p))*G(pTO)*B}{1-pTO}}$$

Otherwise:

W(max) is the configured maximum window size.

$$f(W(\max), pTD, pTO, R, B) = \frac{\frac{1-pTD}{pTD} + W(\max) + \frac{Q(pTO, W(pTD))}{1-pTO}}{R(\frac{W(\max)}{4} + \frac{1-pTD}{pTD*W(\max)} + 2) + \frac{Q(pTO, W(pTD))*G(pTO)*B}{1-pTO}}$$

where:

$$Q(pTO, W(pTD)) = \min(1, (1 - \frac{(1-(1-pTO)^3)(1+(1-pTO)^3(1-(1-pTO)^{w(pTD)-3}))}{1-(1-pTO)^{W(pTD)}})$$

$$G(pTO) = 1 + pTO + 2pTO^2 + 4pTO^3 + 8pTO^4 + 16pTO^5 + 32pTO^6$$